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Eigenvalues for a class of non-Hermitian tetradiagonal Toeplitz matrices

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Abstract and Figures

We study a family of non-Hermitian tetradiagonal Toeplitz matrices having a limiting set consisting of one analytic arc only. We derive individual asymptotic expansions for all eigenvalues as the matrix size grows to infinity. Additionally, we provide specific expansions for the extreme eigenvalues, which are those approaching the endpoints of the limiting set. Although this family does not belong to the simple-loop class, we managed to extend the existing theory to this case. Our results reveal the intricate details of the eigenvalue structure and allow a high accuracy direct calculation.

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tetradiagonal Toeplitz matrices

Manuel Bogoya, Juanita Gasca, and Sergei M. Grudsky

Abstract. We study a family of non-Hermitian tetradiagonal Toeplitz matrices having a limiting set consisting of one analytic arc only. We derive individual asymptotic expansions for all eigenvalues as the matrix size grows to infinity. Additionally, we provide specific expansions for the extreme eigenvalues, which are those approaching the endpoints of the limiting set. Although this family does not belong to the simple-loop class, we managed to extend the existing theory to this case. Our results reveal the intricate details of the eigenvalue structure and allow a high accuracy direct calculation.

1. Introduction

For an absolutely integrable function a over the complex unit circle $\mathbb{T} = \partial\mathbb{D}$, we denote by $T_n(a)$ the $n \times n$ Toeplitz matrix $(a_{j-k})_{j,k=0}^{n-1}$, where a_k represents the k th Fourier coefficient of a , that is

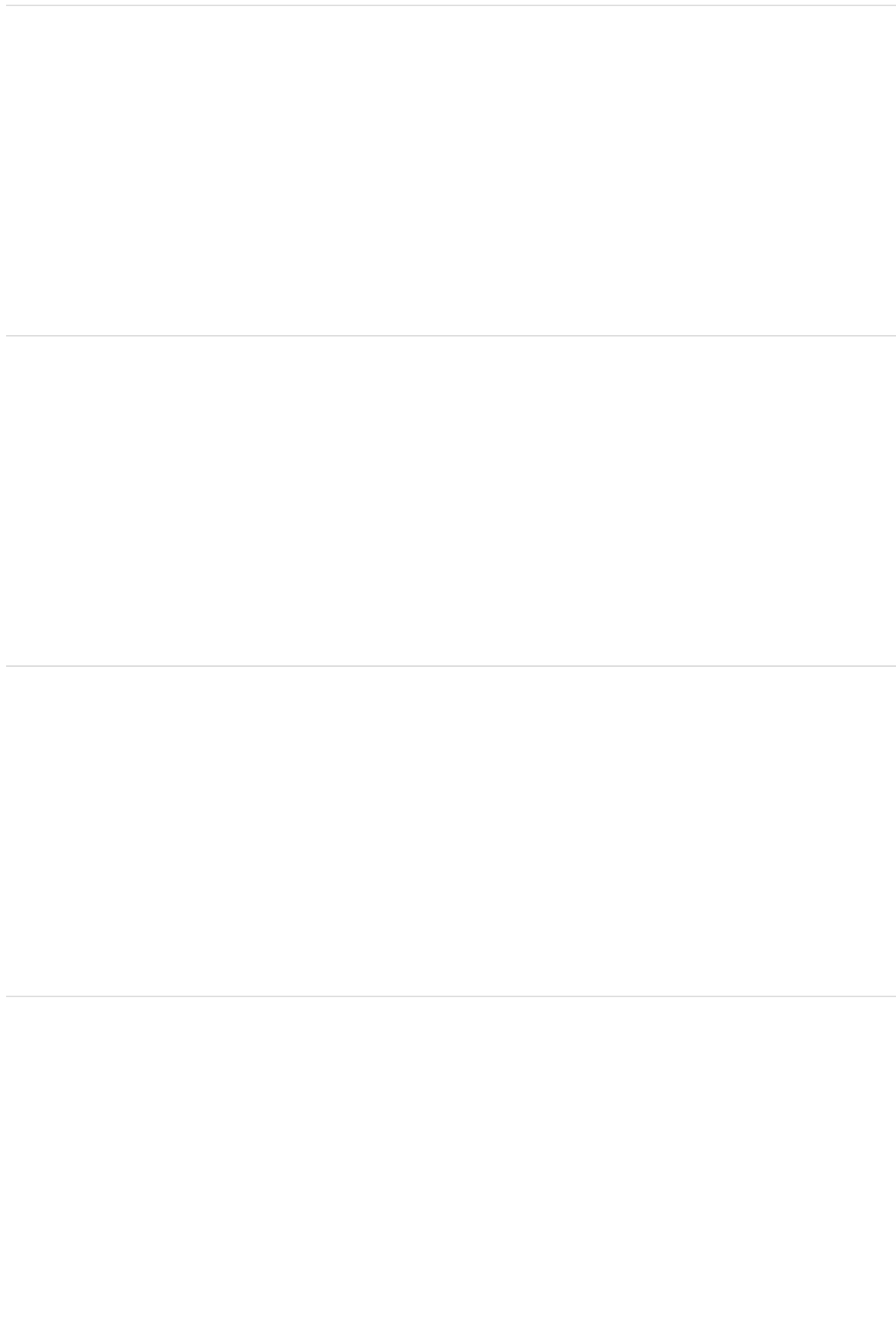
$$a_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} a(e^{i\sigma}) e^{-ik\sigma} d\sigma.$$

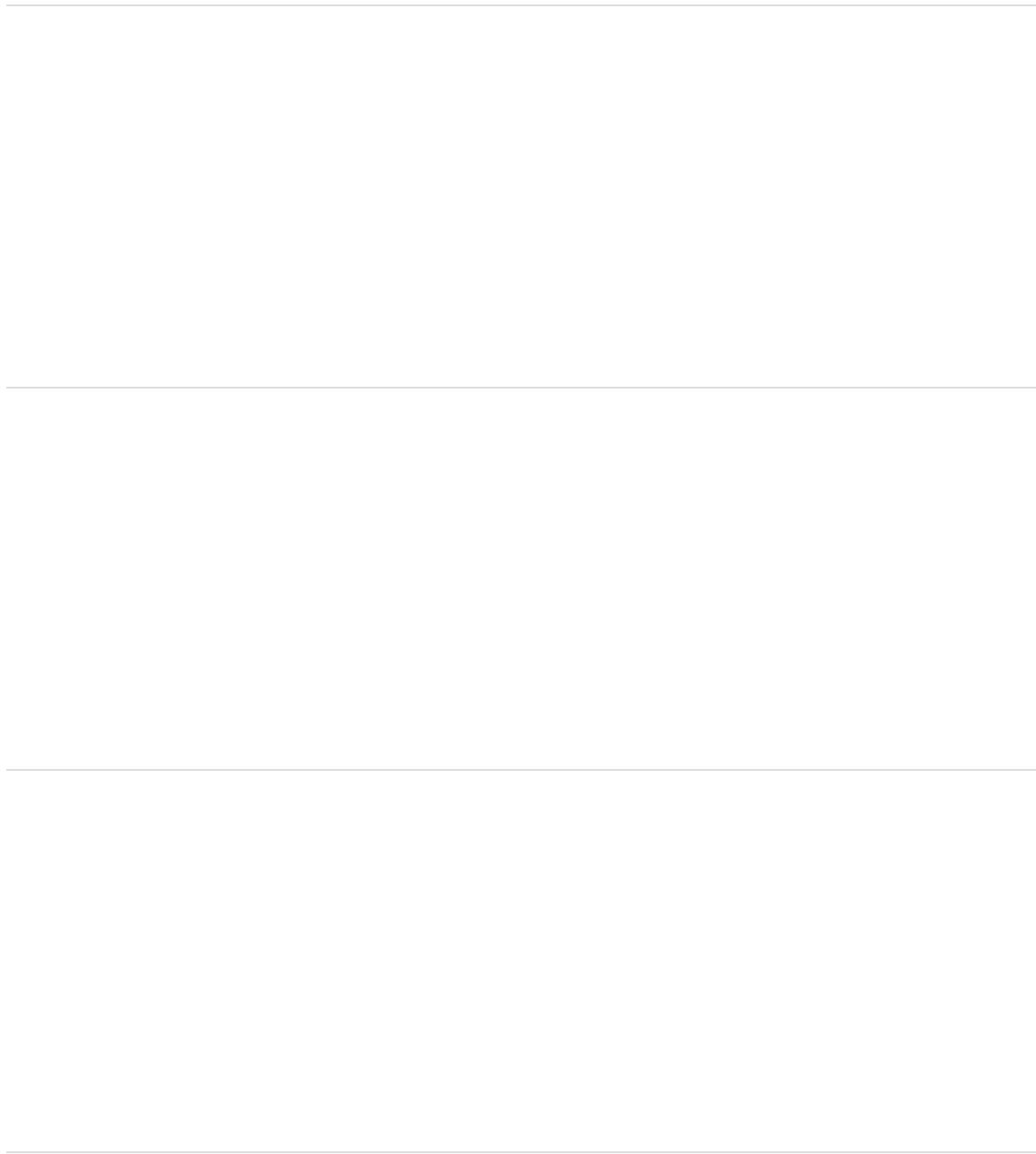
The matrix $T_n(a)$ is characterized by having constant entries along the main diagonals, and the function a is customarily called *symbol* or *generating function*.

The study of Toeplitz matrices started in the first decade of the 20th century with the classic works of O. Toeplitz and G. Szegő (see [15, 16] for the seminal papers or the book [11] for reviewing the classic works, see also [14]). Since then, these matrices have enjoyed popularity because of its many applications, including numerical analysis, engineering, stochastic processes, time series analysis, signal processing, quantum and statistical mechanics, and image processing. Nowadays, a very popular application is the numerical approximation for the solution to differential and integral

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Keywords: Toeplitz matrix, asymptotic expansion, eigenvalues, simple loop method.





Citations (3)

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... There, it was proven that the set of all tetradiagonal Toeplitz matrices is divided into three classes, each of which has the limit set consisting of one, two, or three analytic arcs, respectively. Based on these results, asymptotic expansions of all eigenvalues were obtained in [8, 9] in the case of a limit set consisting of one arc.

...

... The curve (2.5) is shown in Fig. 1. The case where (a) consists of only one analytic arc was considered by Bogoya, Gasca, and Grudsky in [8, 9]; they provide a complete description of the eigenvalue asymptotic expansion of $T_n(a)$ as $n \rightarrow \infty$

... In this paper, we study the individual behavior of the eigenvalues of $n \times n$ Toeplitz matrices $T_n(a)$ as $n \rightarrow \infty$, where a is given by (2.2) with the parameter c in (2.5). We remark that, we use ideas and techniques similar to the ones employed in [8, 9]. Γ is the union of two analytic arcs, one that initiates at $\mu_1 \equiv a(t_1)$ and terminates at $\mu_2 \equiv a(t_2)$, and the second going from $\mu_3 \equiv a(t_3)$ to μ_2 ; moreover, these arcs form a cusp at μ_2 . Figure 2 shows the limit set for some parameter c in $\mathbb{R}$, notice the skeleton shape with respect to the range of a restricted to the unit circle T

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... Note that in [46] the eigenvalues are ordered differently and that our functions f_k are connected with the functions r_k of [46] by the formula $f_k(\theta) = r_k(\pi - \theta)$. In [49], this is generalized to the case of Theorem 8(d),

where (a) is a single arc. ...

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