International Workshop

TOEPLITZ-LIKE OPERATORS AND RELATED TOPICS

Department of Mathematics, CINVESTAV

Mexico City, Mexico

November 18–21, 2008

This year the anual Seminar "Análisis: Norte – Sur" will be held as the International Workshop on TOEPLITZ-LIKE OPERATORS AND RELATED TOPICS. The main goal of the Workshop is to get together leading world experts in the field to discuss the modern state of the theory of Toeplitz and related operators, as well as their numerous applications.

In addition, the Workshop will be held in honor of Nikolai Vasilevski, Professor of the Department of Mathematics, CINVESTAV, on occasion of his 60th birthday.

The Workshop is sponsored by:

- Department of Mathematics, CINVESTAV
  http://www.math.cinvestav.mx/
- Mexican Mathematical Society
  http://www.smm.org.mx/wordpress/

The Organizing Committee of the Workshop is formed of

- Prof. Ronald Douglas, Texas A & M University, College Station, TX, USA,
- Prof. Sergei Grudsky, CINVESTAV, Mexico,
- Dr. Maribel Loaiza Leyva, CINVESTAV, Mexico,
- Prof. Enrique Ramirez de Arellano, CINVESTAV, Mexico.

Schedule

9:00 – 9:50	*Registration* *Opening*	Lewis Coburn	Roland Duduchava	Ilya Spitkovsky
10:00 – 10:50	Ronald Douglas	Nikolai Vasilevski	Vladimir Rabinovich	Yuri Karlovich
Coffee				
11:20 – 12:10	Vladimir Peller	Sergei Grudsky	Nikolai Tarkhanov	Eugene Shargorodsky
12:20 – 13:10	Salvador Pérez Esteva	Raúl Quiroga Barranco	*Lunch*	Michael Shapiro
	Lunch	*Lunch*	14:00 – 19:00 *Excursion*	*Lunch*
15:00 – 15:25	Luís Castro	Maribel Loaiza Leyva		Luis Manuel Tovar
15:30 – 15:55	Vladislav Kravchenko	Josué Ramírez Ortega		Aleksandr Karelin
	Coffee	*Coffee*		*Coffee*
16:20 – 16:45	Tamara Tararykova	Ján Haluška		Slaviša Djordjević
16:50 – 17:15	Carlos Villegas Blas	R. Michael Porter		Valeri Kucherenko
	18:00 – 20:00 *Welcome Cocktail*			19:00 – 21:00 *Conference Dinner*

Workshop program

November 18

8:45 – 9:25 **Registration**

9:30 – 9:55 **Opening**

10:00 – 10:50 **Ronald Douglas**
Multiplication operators on Bergman space

Coffee

11:20 – 12:10 **Vladimir Peller**
Approximation by analytic matrix functions in L_p

12:20 – 13:10 **Salvador Pérez Esteva**
A space of projections on the Bergman space

Lunch

15:00 – 15:25 **Luís Castro**
Invertibility of Toeplitz-like operators arising in oblique derivative boundary value problems for the Helmholtz equation

15:30 – 15:55 **Vladislav Kravchenko**
New representation for solutions of the Sturm-Liouville equation

Coffee

16:20 – 16:45 **Tamara Tararykova**
On Hardy-Steklov and geometric Steklov operators

16:50 – 17:15 **Carlos Villegas Blas**
Asymptotics of clusters of eigenvalues for suitable perturbations of the hydrogen atom

18:00 – 20:00 Welcome Cocktail

November 19

9:00 – 9:50 **Lewis Coburn**
Berezin's operator calculus: regularity estimates and applications

10:00 – 10:50 **Nikolai Vasilevski**
Commutative algebras of Toeplitz operators in action

Coffee

11:20 – 12:10 **Sergei Grudsky**
Dynamic properties of Toeplitz operators on the weighted Bergman spaces

12:20 – 13:10 **Raúl Quiroga Barranco**
Geometry of commutative algebras of Toeplitz operators on the unit ball

Lunch

15:00 – 15:25 **Maribel Loaiza Leiva**
On super Toeplitz operators with radial symbols

15:30 – 15:55 **Josué Ramírez Ortega**
Reproducing kernels of weighted poly-Bergman spaces on the upper half-plane

Coffee

16:20 – 16:45 **Ján Haluška**
Toeplitz operators as vector integrals: distribution symbols

16:50 – 17:15 **R. Michael Porter**
Numerical soluton of Sturm-Liouville spectral problems

November 20

9:00 – 9:50 **Roland Duduchava**
Differential operators on hypersurfaces and shell theory

10:00 – 10:50 **Vladimir Rabinovich**
Essential spectra and exponential estimates of eigenfunctions of discrete spectra of lattice operators of quantum mechanics

Coffee

11:20 – 12:10 **Nikolai Tarkhanov**
The Laplace-Beltrami Operator on a rotationally symmetric surface

Lunch

14:00 – 19:00 Excursion

November 21

9:00 – 9:50 **Ilya Spitkovsky**
On some old and new(er) developments in the theory of algebras generated by two projections

10:00 – 10:50 **Yuri Karlovich**
Wiener-Hopf operators with slowly oscillating matrix symbols on weighted Lebesgue spaces

Coffee

11:20 – 12:10 **Eugene Shargorodsky**
Toeplitz operators with bounded measurable coefficients

12:20 – 13:10 **Michael Shapiro**
On hyperholomorphic Toeplitz operators and Bergman spaces

Lunch

15:00 – 15:25 **Luis Manuel Tovar**
Hyperbolic weighted Bergman classes

15:30 – 15:55 **Aleksandr Karelin, Anna Tarasenko**
Applications of the operator equalities

Coffee

16:20 – 16:45 **Slaviša Djordjević**
The invariant subspaces and spectral properties of linear operators

16:50 – 17:15 **Valeri Kucherenko**
Hyperbolic systems with multiple characteristics

19:00 – 21:00 Conference Dinner

Abstracts

Luís P. Castro

Invertibility of Toeplitz-like operators arising in oblique derivative boundary value problems for the Helmholtz equation

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We will consider Toeplitz-like operators arising in a class of boundary value problems in weak formulation where two boundary conditions are given on the half-lines bordering the first quadrant that contain oblique derivatives. The boundary pseudodifferential operators which characterize the corresponding problems for the Helmholtz equation are reduced by matricial coupling relations to certain compositions of Wiener-Hopf and even and odd extension operators which will be analyzed in detail. Results are: explicit construction of (lateral) inverses in Bessel potential spaces, eventually after normalization, and regularity results. Part of the talk is based on joint work with F.-O. Speck and F. S. Teixeira.

Lewis A. Coburn

Berezin's operator calculus: regularity estimates and applications

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F. A. Berezin introduced a general "symbol calculus" for linear operators on reproducing kernel Hilbert spaces. For the Hilbert space $H^2(\mathbf{C}^n, d\mu)$ of Gaussian square-integrable entire functions on complex n-space, $\mathbf{C}^n$, as well as for the Bergman Hilbert spaces $A^2(\Omega)$ of Euclidean volume square-integrable holomorphic functions on Ω, an arbitrary bounded domain in $\mathbf{C}^n$, I obtained sharp Lipschitz estimates for Berezin symbols of arbitrary bounded operators. More recently, my student Bo Li and I obtained the corresponding

sharp directional derivative estimates. These results and some applications and recent extensions are discussed.

Main references. There are five papers in the *Proceedings of the American Mathematical Society*: (1985) Mazur, Pflug and Skwarczynski; (2005) Coburn; (2006) Engliš and Zhang; (2007) Coburn; (2008) Coburn and Li.

Slaviša V. Djordjević

The invariant subspaces and spectral properties of linear operators

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For a linear bounded operator $T \in B(X)$ and its invariant closed subspace $E \subset X$, with $T_{|E}$ we denote the restriction of T to the invariant subspace E and with $T_{|X/E}$ the mapping determined by T on the quotient space X/E of this invariant subspace. In this talk we will show that for an operator $T \in B(X)$:

(a) if $T_{|E}$ has closed range, then T is invertible if and only if $T_{|E}$ is bounded below, $T_{|X/E}$ is onto and $N(T_{|X/E})$ is isomorphic to $E/R(T_{|E})$;

(b) $\sigma(T_{|E}) \cup \sigma(T_{|X/E}) = \sigma(T) \cup \{S((T_{|E})^*) \cap S(T_{|X/E})\} = \sigma(T) \cup \{\sigma(T_{|E}) \cap \sigma(T_{|X/E})\}$.

It is known that for an operator $T \in B(X)$, if any two of T, $T_{|E}$ and $T_{|X/E}$ are Fredholm, then so is the third one: we prove that the Fredholm spectrum $\sigma_e(T)$ of T, $T_{|E}$ and $T_{|X/E}$ satisfy the equality $\sigma_e(T_{|E}) \cup \sigma_e(T_{|X/E}) \cup \{S((T_{|E})^*) \cap S(T_{|X/E})\} = \sigma_e(T) \cup \{S((T_{|E})^* \cap S(T_{|X/E})\}$. The Browder spectrum $\sigma_b(\cdot)$ satisfies a more satisfactory property: we prove that $\sigma_b(T_{|E}) \cup \sigma_b(T_{|X/E}) = \sigma_b(T) \cup \{S((T_{|E})^*) \cap S(T_{|X/E})\}$. The relationship between the Weyl spectra of T, $T_{|E}$ and $T_{|X/E}$ is a bit more delicate: it is proved that $\sigma_w(T_{|E}) \cup \sigma_w(T_{|X/E}) \subseteq \sigma_b(T) \cup \{S((T_{|E})^*) \cap S(T_{|X/E})\} \subseteq \sigma_w(T) \cup \{S_e(P) \cup S(Q)\}$, where either $P = T_{|E}$ and $Q = T_{|X/E}$ or $P = (T_{|X/E})^*$ and $Q = (T_{|E})^*$.

Ronald G. Douglas

Multiplication operators on Bergman space

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In joint work with S. Sun and D. Zheng we study the reducing subspaces for multiplication operators on Bergman space defined by a finite Blaschke product $B(z)$. This builds on work by a number of researchers pursuing a question posed by K. Zhu.

One shows the relation of the von Neumann algebra generated by projections on to reducing subspaces to the Riemann surface S_B defined by $B(z)$. In particular, we show that the dimension of this algebra is equal to the number of connected components of S_B and show if the number of zeros is eight or less that the algebra is commutative. These results provide a complete description in the latter cases but we leave unresolved the question of whether the algebra is commutative in general. The emphasis in this talk will be on a conceptual understanding of the structure revealed by our results rather than on the technical details.

Roland Duduchava

Differential operators on hypersurfaces and shell theory

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Partial differential equations on hypersurfaces and the corresponding boundary value problems, are encountered rather often in applications and are often used to prepare ground for an investigation of a shell equation.

There exist a number of approaches proposed for modeling linearly elastic flexural shells (see papers by Cosserats (1909), Goldenveiser (1961), Naghdi (1963), Vekua (1965), Novozhilov (1970), Koiter (1970)). We suggest a different approach based on a calculus of Günter's and Stock's derivatives on a hypersurface $\mathscr{S}$ given by a local immersion $\Theta\colon \Omega \to \mathscr{S} \subset \mathbb{R}^n$, $\Omega \subset \mathbb{R}^{n-1}$. Differential operators on the surface are represented in

terms of Günter's derivatives $\mathscr{D}_j := \partial_j - \nu_j(\mathscr{X})\partial_\nu$, $j = 1, \dots, n$, where $\nu = (\nu_1, \dots, \nu_j)^\top$ is the outer unit normal vector to $\mathscr{S}$ and $\partial_\nu := \sum_{j=1}^n \nu_j \partial_j$ is the normal derivative. We write the surface gradient $\nabla_{\mathscr{S}}$, the surface divergence $\mathrm{Div}_{\mathscr{S}}$, the Laplace-Beltrami operator $\mathbf{\Delta}_{\mathscr{S}}$, the Lamé operator $\mathscr{L}_{\mathscr{S}}$, the deformation tensor $\mathrm{Def}_{\mathscr{S}}(U)$, the covariant derivative $\nabla^{\mathscr{S}}_W U$ (the Levi-Civita connection) in terms of Günter's derivatives in rather compact form, very similar to their representation in cartesian coordinates of the ambient Euclidean space $\mathbb{R}^n$.

Concerning an application to shell theory: a shell is thought as a "tubular" neighborhood

$$\Omega^\varepsilon := \left\{ x \in \mathbb{R}^n \colon x = \mathscr{X} + t\boldsymbol{\nu}(\mathscr{X}) = \Theta(y) + t\boldsymbol{\nu}\big(\Theta(y)\big), \quad y \in \omega, \quad -\varepsilon \le t \le \varepsilon \right\}$$

around the middle surface $\mathscr{S}$. It seems rather natural to use a natural coordinate system-projection of canonical vectors $\boldsymbol{d}^j = \pi_{\mathscr{S}} \boldsymbol{e}^j$, where $\boldsymbol{e}^1 = (1, 0, \dots, 0)$, $\dots$, $\boldsymbol{e}^n = (0, 0, \dots, 1)$ is the natural basis in the Euclidean space $\mathbb{R}^n$. Extending the chosen system by a unit normal vector $\boldsymbol{d}^{n+1} = \boldsymbol{\nu}$ to the mid-surface $\mathscr{S}$, we get a linearly dependent but full system of unit vectors in a "tubular" neighborhood Ω^ε, given by the immersion $\Omega^\varepsilon \colon \omega^\varepsilon := \omega \times [-\varepsilon, \varepsilon] \to \Omega^\varepsilon$, $\omega^\varepsilon \subset \mathbb{R}^n$, where $\Theta^\varepsilon(y, t) := \Theta(y) + t\boldsymbol{\nu}(y)$ $(y, t) \in \omega^\varepsilon$, provided ε is small. The new system $\left\{\boldsymbol{d}^j\right\}_{j=1}^{n+1}$ is full but linearly dependent. It turned out that the classical operators, such as divergence, gradient, Laplacian, Lamé and a related deformation tensor, written in the new system in the tubular domain Ω^ε, can be restricted to the middle surface $\mathscr{S}$ and coincide there with the corresponding operators written in terms of Günter's derivatives. this is a first but essential step to the application of asymptotic methods.

The above mentioned operators and, specially, the boundary value problems for Lamé equations, written in the new curvilinear system, are investigated in detail. For this purpose we apply essentially Korn's inequalities, which are proved with the help of Lions's lemma. We generalize this lemma, proved earlier for $\mathbb{L}_2$-based Sobolev spaces, to $\mathbb{L}_p$-based Bessel potential spaces for arbitrary $1 < p < \infty$.

Sergei Grudsky

Dynamic properties of Toeplitz operators on the weighted Bergman spaces

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The spectral theory of commutative C^*-algebras of Toeplitz operators on Bergman spaces was constructed and developed by Nikolai Vasilevskii and his coauthors during the last decade. For each such commutative algebra it has been constructed a unitary operator which reduce each operator from this algebra to a certain multiplication operator. This gives us a powerful research tool allowing direct access to study many properties of Toeplitz operators.

The lecture is devoted to "dynamic" properties of such operators. More exactly we investigate boundedness, compactness, spectral properties etc., of the Toeplitz operators as depending on power weights in the Bergman space. These investigations are motivated by the Berezin quantization procedure.

The present lecture is based on the joint work with Nikolai Vasilevski and Alexey Karapetyants.

Ján Haluška

Toeplitz operators as vector integrals: distribution symbols

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The theory of bilinear vector integration might be useful in the theory of integral Toeplitz operators since it opens new directions of investigations. But it is not a one-direction way, it holds also vice-versa. The usefulness of considering of Toeplitz operators as vector integrals for the Lebesgue measure and integral theory consists of two items: 1. it is a way to equip some Lebesgue integration with geometry directly in the integral construction, where 2. three basic integration structures (set, measure, function) are organically bounded into one whole.

I. Dobrakov built up a Lebesgue-type integration theory for X-valued functions and operator $L(X,Y)$-valued measure σ-additive in the strong operator topology, where X, Y are Banach spaces (real or complex). This theory completely generalizes the theory described in the well-known "Measure theory" book of P. Halmos. The convergence of integrable functions is pointwise in the limit integration theorems. The important real quantity in the Dobrakov integration theory is a subadditive set function called semivariation (of the operator measure). This semivariation need not be continuous in the construction of integral and this fact distinguishes Dobrakov's integration from other strong vector integration theories, because this implies that the space of integrable functions in the sense of Dobrakov is a proper superset of spaces of integrable functions in the sens of Bartle, Dunford-Schwarz, Bochner, etc., when considering the same settings. Some theorems also hold which have no scalar analogues. The complete generalization of Dobrakov's integration from Banach spaces to X, Y being the inductive limits of Banach spaces was done by the author. Spaces which are inductive limits of Banach spaces involve, in particular, the well known Schwarz test and distribution function spaces.

Present state of the investigation: We solved problems about applicability of the strong bilinear integration theory to Toeplitz operators in the basic situation (unit disc, Bergman space). No additional conditions are needed. We extended the notion of Toeplitz operator to symbol functions as distributions. We overtake some theorems from the strong operator integration theory to Toeplitz integral operator theory, e.g. directly from the construction we obtain a representation theorem (the generalized Toeplitz operator = integral with respect to an operator valued measure), and some others. The computational question, how to deal with the generalized Toeplitz operators, was solved using distributions in the form of Antosik-Mikusinski-Sikorski, i.e. considering distributions as equivalent classes of sequences of test functions.

References

[1] Antosik P., Mikusiński J., Sikorski R.: Theory of Distributions. Elsevier, Amsterdam 1973.

[2] Dobrakov I.: On integration in Banach spaces I., Czechoslovak Math. J. **20** (1970), 511–536 (and parts No. II-XVIII).

[3] Haluška J.: On integration in complete bornological locally convex topological vector spaces. Czechoslovak Math. J. **47** (1997), 205–219.

[4] Vasilevski N. L.: Commutative Algebras of Toeplitz Operators on the Bergman Space. Birkhäuser, Basel–Boston–Berlin, 2008.

Aleksandr Karelin, Anna Tarasenko

Applications of the operator equalities

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We denote the Cauchy singular integral operator along a contour Γ by

$$(S_\Gamma\varphi)(t) = \frac{1}{\pi i}\int\limits_\Gamma \frac{\varphi(\tau)}{\tau - t}d\tau$$

and the identity operator on Γ by $(I_\Gamma\varphi)(t) = \varphi(t)$.

Suppose that

$$A = aI_\Gamma + bZ_\Gamma + cS_\Gamma + dZ_\Gamma S_\Gamma,$$

where a, b, c, d are bounded measurable functions on Γ and

$$(Z_\Gamma\varphi)(\tau) = \varphi(-\tau).$$

We will refer to the direct relation between the operator A with a model involution and a matrix characteristic singular integral operator D without additional associated operators as operator equalities: for an orientation-preserving shift it is a similarity transform

$$D = FAF^{-1}$$

and for an orientation-reversing shift it is a transform by two invertible operators

$$D = \mathcal{H}A\mathcal{E}.$$

Applications of operator equalities to some scalar singular integral operators with model involutions, Riemann boundary value problems, characteristic matrix singular integral operators are considered.

Yu. I. Karlovich

Wiener-Hopf operators with slowly oscillating matrix symbols on weighted Lebesgue spaces

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Fredholm conditions and an index formula are obtained for Wiener-Hopf operators $W(a)$ with slowly oscillating matrix symbols a on weighted Lebesgue spaces $L^p_N(\mathbb{R}_+, w)$ where $1 < p < \infty$, w is a Muckenhoupt weight in $A_p(\mathbb{R})$ and $N \in \mathbb{N}$. The entries of matrix symbols belong to a Banach subalgebra of Fourier multipliers on $L^p(\mathbb{R}, w)$ that are continuous on $\mathbb{R}$ and have, in general, different slowly oscillating asymptotics at $\pm\infty$. To define the Banach algebra $SO_{p,w}$ of corresponding slowly oscillating functions, we apply the theory of pseudodifferential and Calderón-Zygmund operators. Established sufficient conditions become a Fredholm criterion in the case of Muckenhoupt weights with equal indices of powerlikeness, and also for Muckenhoupt weights with different indices of powerlikeness under some condition on p, w and a.

Vladislav V. Kravchenko

New representation for solutions of the Sturm-Liouville equation

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We present a new representation for solutions of the Sturm-Liouville equation which was obtained in [1]. It gives solutions in the form of Taylor series with respect to the spectral parameter and lends itself to numerical computation. Its applications to initial value and spectral problems are discussed.

[1] V. V. Kravchenko. 2008. Complex Variables and Elliptic Equations **53** issue 8, 775–789.

Valeri V. Kucherenko

Hyperbolic systems with multiple characteristics

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Theorems of uniqueness and existence are proved for the hyperbolic systems and equations with multiplicity too. If the characteristics are non-involutory the low ordered terms may result only in the loss of smoothness of solutions compared with the smoothness of the initial data. But if the multiplicity is more or equal to three and the characteristics are non-involutory, some hyperbolic systems are ill-posed due to effects of low order terms. It can be caused by the trap for the "branching of characteristics" near the points of triple intersection for the characteristics.

Maribel Loaiza Leyva

On super Toeplitz operators with radial symbols

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We study super Toeplitz operators with radial symbols acting on the Bergman space of the super unit disk. We prove that the algebra generated by this kind of operators is commutative.

Vladimir Peller

Approximation by analytic matrix functions in L^p

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The talk is based on joint results with L. Baratchart and F. Nazarov. For $2 \le p < \infty$, we consider a problem of approximating in the norm of L^p a matrix function Φ on the unit circle by matrix functions analytic in the unit disc. It turns out that the space of matrix functions in L^p splits into two massive subsets: the set of respectable functions and the set of weird functions. For respectable $m \times n$ matrix functions Φ the distance to the set of analytic functions is equal to the norm of the Hankel operator H_Φ from $H^q(C^n)$ to $H^2_-(C^m)$, where $1/p + 1/q = 1/2$. For weird functions Φ the distance is greater than the norm of the Hankel operator. We have found another distance formula that works for all matrix functions in L^p. We also consider related factorization formulae, describe all best approximants and characterize badly approximable matrix functions.

Salvador Pérez Esteva

A space of projections on the Bergman space

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We define and study a family of projections on the Bergman space in the disk, parameterized by an affine closed space of a Banach space of holomorphic functions in the disk. This family includes the classical Forelli-Rudin projections. This is joint work with Oscar Blasco.

R. Michael Porter

Numerical solution of Sturm-Liouville spectral problems

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V. Kravchenko has shown how to apply the theory of pseudoanalytic functions of L. Bers and I. Vekua to write an explicit basis for solutions of numerous differential equations. In particular a specific power series is produced whose zeros are the eigenvalues of Sturm-Liouville problems. We show numerical results for a number of standard test cases.

Raúl Quiroga Barranco

Geometry of commutative algebras of Toeplitz operators on the unit ball

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In recent joint work with N. Vasilevski, we have discovered several non-equivalent algebras of Toeplitz operators on the unit ball. These algebras are closely related to certain submanifolds with distinguished geometric features. More specifically, the algebras found are given by pairs of orthogonal foliations, one of which is totally geodesic and the other Riemannian with flat parallel leaves. We will describe the geometric properties of these objects and how they were used in the discovery of commutative algebras of Toeplitz operators.

Vladimir Rabinovich

Essential spectra and exponential estimates of eigenfunctions of discrete spectra of lattice operators of quantum mechanics

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The main aim of the talk is to give the estimates of the exponential decreasing of eigenfunctions of difference operators on the lattice $\mathbb{Z}^n$ which are the discrete analogs of the Schrödinger, Dirac and square-root Klein-Gordon operators.

Note that Schrödinger operators on the lattice $\mathbb{Z}^n$ describe the so-called tight binding models in the solid state physics, the propagation of spin waves and waves in quasi-crystals, mathematical models of the nanostructure, etc. In recent times the discrete relativistic operators also attract much attention because it has been used in comparative studies of relativistic and nonrelativistic electronlocalization phenomena, in relativistic investigations of electrical conduction in disordered systems, in the construction of supertransparent models with supersymmetric structures, and in relativistic tunnelling problems.

The problem of the exponential estimates of the solutions of the elliptic partial differential equations with applications to the Schrödinger operator is classical. Note also our recent papers [1, 2] where we proposed a new approach to exponential estimates for partial differential and pseudodifferential operators, based on the limit operators method (see [3]). A similar approach is proposed in the talk. Our investigation of the essential spectra and the exponential decay of eigenfunctions of the discrete spectra are based on the calculus of the so-called pseudodifference operators (pseudodifferential operators on the group $\mathbb{Z}^n$) with analytic symbols, developed in the paper [4], and the limit operators method (see the book [3] and the references cited there). We give a description of the essential spectrum and the estimates of the eigenfunctions of main lattice operators of the Quantum Mechanics: 1. Matrix Schrödinger operators on $\mathbb{Z}^n$, 2. Dirac operators on $\mathbb{Z}^3$, 3. Square root Klein-Gordon operators on $\mathbb{Z}^n$.

We present results obtained with Steffen Roch.

[1] V. S. Rabinovich, S. Roch, *Essential spectrum and exponential decay estimates of*

solutions of elliptic systems of partial differential equations, Georgian Mathematical Journal, Volume 15 (2008), N. 2, 333–351.

[2] V. S. Rabinovich, S. Roch, *Essential spectra of pseudodifferential operators and exponential decay of their solutions. Applications to Schrödinger operators.* Operator Theory: Advances and Applications, vol. 181, 355–384, 2008, Birkhäuser Verlag, Basel / Switzerland.

[3] V. S. Rabinovich, S. Roch, B. Silbermann, *Limit Operators and its Applications in the Operator Theory*, In ser. Operator Theory: Advances and Applications, vol 150, Birkhäuser Verlag, 2004.

[4] V. S. Rabinovich, S. Roch, *Pseudodifference operators on weighted spaces, and applications to discrete Schrodinger operators*, Acta Applicandae Math. **84**: 55–96, 2004.

Josué Ramírez Ortega

Reproducing kernels of weighted poly-Bergman spaces on the upper half-plane

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Let Π be the upper half-plane. The weighted poly-Bergman spaces on Π consist of all functions on $L_2(\Pi, (\lambda+1)(2y)^\lambda dxdy)$ satisfying the equation $\left(\frac{\partial}{\partial \bar{z}}\right)^n f = 0$. In case $\lambda = 0$ new representations of poly-Bergman kernels are given by differentiation of rational functions. In general, the weighted poly-Bergman kernels are given by means of the action semidirect product of $\mathbb{R}$ and $\mathbb{R}^+$.

Michael Shapiro

On hyperholomorphic Toeplitz operators and Bergman spaces

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The talk pretends to give both the basic notions of the topics referred to in the title and the recent advances in them which include the developments the framework of quaternionic and Clifford analysis as well as some applications to classic areas such as functions of two, and several, complex variables, and Laplacian vector fields.

Eugene Shargorodsky

Toeplitz operators with bounded measurable coefficients

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The talk is intended as a review of some recent progress and open problems in the study of the essential spectra of Toeplitz operators with bounded measurable coefficients. In particular, an optimal sufficient condition for a point to belong to the essential spectrum of a Toeplitz operator will be discussed. The optimality of this condition has been recently established in a joint work with S. Grudsky where the main ingredient is a new sufficient condition for a composition of a Muckenhoupt weight with a Blaschke product to belong to the same Muckenhoupt class.

Ilya Spitkovsky

On some old and new(er) developments in the theory of algebras generated by two projections

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We will start with a well (but for some reason, not widely) known canonical form of a pair $\{P, Q\}$ of orthogonal projections. We will then show how this form can be used to answer a variety of natural questions about operators from the algebra generated by P and Q. In particular, invertibility and Fredholmness criteria, norm formulas and certain polynomial equations in P, Q will be discussed.

Tamara V. Tararykova

On Hardy-Steklov and geometric Steklov operators

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Let a and b be strictly incresing, differentiable functions defined on $[0, \infty)$ satisfying $a(0) = b(0) = 0$, $a(x) < b(x)$ for $x > 0$, and $\lim_{x\to\infty} a(x) = \lim_{x\to\infty} b(x) = \infty$. Consider the Hardy-Steklov operator S defined by

$$(Sf)(x) = \frac{1}{b(x) - a(x)} \int_{a(x)}^{b(x)} f(t)\, dt\,, \quad x > 0\,,$$

and the corresponding geometric Steklov operator G_S defined by

$$(G_S f)(x) = \exp\left(\frac{1}{b(x) - a(x)} \int_{a(x)}^{b(x)} \ln f(t)\, dt\right), \quad x > 0\,, \quad f \geq 0\,.$$

Necessary and sufficient conditions on weight functions u and v ensuring the boundedness of the Hardy-Steklov operator S from $L^p_u(0, \infty)$ to $L^q_v(0, \infty)$ where $1 < p \leq q < \infty$ were

obtained in 1998 by H. Heinig and G. Sinnamon. Also a two sided estimate for the norm of this operator was given.

In this talk we shall present necessary and sufficient conditions on u and v of different (equivalent) form. Importantly the accompanying estimates for the norm of S are sharper and are such that, in contrast to the estimates obtained by the aforementioned authors, they allow obtaining by using a limiting procedure sufficient conditions on u and v ensuring the boundedness of the geometric Steklov operator S_G from $L^p_u(0,\infty)$ to $L^q_v(0,\infty)$ where $0 < p \le q < \infty$, which for some a and b coincide with necessary ones.

Nikolai Tarkhanov

The Laplace-Beltrami Operator on a Rotationally Symmetric Surface

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The aim of this work is to highlight a number of analytic problems which make the analysis on manifolds with true cuspidal points much more difficult than that on manifolds with conic points while such singularities are topologically equivalent. To this end we discuss the Laplace-Beltrami operator on a compact rotationally symmetric surface with a complete metric. An experienced reader can readily interpret this in a general framework of the calculus on compact manifolds with cusps which is due to V. Rabinovich et al. (1997).

Luis Manuel Tovar

Hyperbolic weighted Bergman classes

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In this paper we continue our studies on hyperbolic weighted Bergman classes by Green's function and Möbius transformations. Likewise we study its series expansions and how to define a metric in such way, that these classes result in a convex complete metric space.

Nikolai Vasilevski

Commutative algebras of Toeplitz operators in action

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We describe first the commutative C^*-algebras generated by Toeplitz operators acting on the Bergman space over the unit disk. Surprisingly their complete characterization can be done in terms of pencils of geodesics on the disk equipped with the hyperbolic metric. Specific properties of Toeplitz operators from the above commutative algebras permit us to attack successfully a number of deep problems in the area.

We discuss the following three of them:

– difference between compactness of commutators and semi-commutators of Toeplitz operators;

– fine structure of the algebra generated by Toeplitz operators with piece-wise continuous symbols;

– commutative algebras of Toeplitz operators on bounded symmetric domains.

Carlos Villegas Blas

Asymptotics of clusters of eigenvalues for suitable perturbations of the hydrogen atom

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Let us consider the Schrödinger Hamiltonian operator $H_\hbar = H_0 + \epsilon Q_\hbar$ where H_0 is the Hamiltonian of the hydrogen atom (with the Planck parameter $\hbar$ included), $Q_\hbar$ is a pseudodifferential operator of order zero uniformly bounded in $\hbar$ and $\epsilon = O(\hbar^\delta)$ with $\delta > 1$. We show that by taking $\hbar = 1/N$ with N a natural number, the family of operators $H_{1/N}$ has well defined clusters of eigenvalues around the number $-1/2$. We

obtain the limiting eigenvalue distribution of such clusters when N goes to infinity. The result involves the averages of the principal symbol of $Q_\hbar$ along the classical orbits with fixed energy $E = -1/2$ in the phase space of the Kepler problem. The collision orbits are considered in our analysis. A Szegö-type theorem is proved as part of the proof of the main theorem and we also make use of a suitable set of coherent states.