International Workshop on

TRANSMUTATION OPERATORS

AND

RELATED TOPICS

I_WTORT 2019

September 17–18, 2019

Cinvestav, Querétaro, México

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Conference Program

Tuesday, September 17, 2019

Tuesday (Auditorium)

10:00–11:00	REGISTRATION
11:00–11:10	OPENING CEREMONY

Chair: V. Kravchenko

11:10–12:00	SERGEY SITNIK Historical survey of transmutation theory: persons and results
12:10–13:00	VLADIMIR RABINOVICH One-dimensional Dirac operators with delta-interactions
13:00–13:20	SERGII TORBA An improved NSBF representation for regular solution of perturbed Bessel equations

Tuesday (Dining Area)

13:20	L U N C H

Tuesday (Auditorium)

Chair: S. Torba

14:20–15:10	RAFAEL DEL RÍO Resonances under Rank One Perturbations
15:10–16:00	SERGEI GRUDSKY Soliton theory and Hankel operators
	B R E A K

Chair: R. del Río

16:20–17:10	VLADISLAV KRAVCHENKO A method for practical solution of forward and inverse spectral problems on finite and infinite intervals
17:10–18:00	LUIS O. SILVA Sampling, oversampling, and subsampling in de Branges spaces: recent results

18:00–18:20	ULISES VELASCO Non-Linear Fourier transform using transmutation operators and SPPS representations
18:20–18:40	FRANCISCO EDUARDO URBANO ALTAMIRANO Research of dispersion problem in quantum waveguides by means of spectral parameter power series

Wednesday, September 18, 2019

Wednesday (Auditorium)

Chair: V. Barrera

9:30–10:20	ELINA SHISHKINA Method of Riesz potentials applied to solution to nonhomogeneous singular wave equations
10:20–11:10	SERGEY SITNIK Historical survey of transmutation theory: persons and results
	B R E A K

Chair: M. Porter

11:30–12:10	VÍCTOR BARRERA-FIGUEROA On the spectra of one-dimensional Schrödinger operators with interactions on $\mathbb{R}$
12:10–12.50	RAÚL CASTILLO PÉREZ A method for computation of scattering amplitudes and Green functions of whole axis problems
12:50–13:10	MAHMOUD ABUL-EZ On Polynomial Bases Theory in the Context of Clifford Analysis and its Applications

Wednesday (Outdoors)

13:10	CONFERENCE PHOTO

Wednesday (Dining Area)

13:20	L U N C H

Wednesday (Auditorium)

Chair: R. Castillo

14:20–15:10	JOAO MORAIS Approximation of Band-limited Functions by Quaternionic Ball Prolate Spheroidal Wave Signals
15:10–15:30	R. MICHAEL PORTER Perturbed Bessel equation with several spectral parameters
15:30–15:50	VÍCTOR ALFONSO VICENTE BENÍTEZ Transmutation operators and complete systems of solutions for the d-dimensional radial Schrödinger equation
	BREAK

Chair: J. Morais

16:10–16:30	SAMANTHA ANA CRISTINA LOREDO RAMÍREZ On the Green's function of a quantum waveguide with perturbations
16:30–16:50	LETICIA OLIVERA RAMÍREZ Spectral analysis of one-dimensional Schrödinger operators with two point interactions
16:50–17:10	FLOR ALBA GÓMEZ GOMEZ Neumann series of Bessel functions representations for solutions of linear higher order differential equations
	BREAK

Chair: E. Shishkina

17:20–17:40	LUIS ENRIQUE GEN ROMERO Analytical and numerical analysis of waveguides and photonic crystals using the SPPS method
17:40–18:00	RAYBEL GARCÍA ANCONA Contragenic Functions on Spheroidal Domains
18:00–18:20	PABLO ENRIQUE MOREIRA GALVÁN Runge theorem for solutions of $(D + M^\alpha)\varphi = 0$
18:20–18:30	CLOSING PROGRAM

Abstracts of Talks

On Polynomial Bases Theory in the Context of Clifford Analysis and its Applications

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This talk focuses on the expansion of Clifford-valued functions into bases of special monogenic polynomials. In this sense, selected research work is embedded in the main stream of classical complex analysis, which was originally founded by J.M. Whittaker and B. Cannon. A recent research domain is broadened by studying the problem of bases of polynomials in the context of special classes of functions in Clifford analysis. All familiar sets of complex polynomials e.g. those of Legendre, Laguerre, Hermite, Bernoulli form simple bases. In this talk, we introduce applications to important special functions related to Bernoulli and Bessel Polynomials in the Clifford analysis setting.

On the spectra of one-dimensional Schrödinger operators with interactions on $\mathbb{R}$

VÍCTOR BARRERA-FIGUEROA, VLADIMIR S. RABINOVICH
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This talk is devoted to study the spectral properties of one-dimensional Schrödinger operators

$$S_q u(x) = \left(-\frac{d^2}{dx^2} + q(x)\right) u(x), \quad x \in \mathbb{R}, \tag{1}$$

with potentials $q = q_0 + q_s$, where $q_0 \in L^\infty(\mathbb{R})$ is a regular potential, and $q_s \in \mathcal{D}'(\mathbb{R})$ is a singular potential supported on an infinite discrete set $\mathcal{Y} \subset \mathbb{R}$. We consider a self-adjoint extension $\mathcal{H}$ of the formal operator (1) given by an unbounded operator in $L^2(\mathbb{R})$ defined by the Schrödinger operator S_{q_0} involving only the regular potential q_0, and certain interaction conditions at the points of $\mathcal{Y}$. We study the spectral properties of the operator $\mathcal{H}$ under general conditions by means of limit operators, and the important case when the set $\mathcal{Y}$ has a periodic structure. For this case we employ the Spectral Parameter Power Series method for determining the band-gap spectra of periodic operators in terms of monodromy matrices.

A method for computation of scattering amplitudes and Green functions of whole axis problems

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A method for the computation of scattering data and of the Green function for a one-dimensional Schrodinger operator H with a decaying potential $q(x)$ is presented. It is based on representations for the Jost solutions in the case of a compactly supported potential obtained in terms of Neumann series of Bessel functions (NSBF). The representations are used for calculating a complete orthonormal system of generalized eigenfunctions of the Schrodinger operator, which in turn allow one to compute the scattering amplitudes and the Green function of the operator $H - \lambda$ with $\lambda \in \mathbb{C}$.

Resonances under Rank One Perturbations

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This talk is about resonances generated by rank one perturbations of self-adjoint operators with eigenvalues embedded in the continuous spectrum. Instability of these eigenvalues is analyzed and almost exponential decay for the associated resonant states is exhibited. These results can be applied to Sturm-Liouville operators. Main tools are the Aronszajn-Donoghue theory for rank one perturbations, a reduction process of the resolvent based on Feshbach-Livsic formula, the Fermi golden rule and a careful analysis of the Fourier transform of quasi-Lorentzian functions. This is joint work with Olivier Bourget, Víctor H. Cortés and Claudio Fernández from the Pontificia Universidad Católica de Chile.

Contragenic Functions on Spheroidal Domains

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A well-known result in complex analysis says that every harmonic function $u\colon D(0;1) \to \mathbb{C}$, where $D(0;1)$ denotes the unit disk, is expressible as the sum of a holomorphic function and an antiholomorphic function. This result has many applications; for example, it is possible to develop methods, such as the Fornberg method, to find conform mappings defined on $D(0;1)$. On the other hand, there are many generalizations for monogenic functions in the algebra of quaternions $\mathbb{H}$, Clifford algebras and monogenic functions from $\mathbb{R}^3$ to $\mathbb{H}$. However, the natural generalization for monogenic functions from $\mathbb{R}^3$ to $\mathbb{R}^3$ is not satisfied [1]. Consequently, it is possible to find harmonic functions that are orthogonal to the system of monogenic and antimonogenic functions in the sense of L_2, called contragenic functions. In this talk, based on techniques and results studied in [3], contragenic polynomials for spheroids of the form $x^2 + (y^2 + z^2)/e^{\nu}, \nu \in \mathbb{R}$ will be constructed.

[1] C. ÁLVAREZ-PEÑA, R. M. PORTER, (2013), *Contragenic functions of three variables,* Complex Anal. Oper. Theory. 7:1.

[2] R. GARCÍA A., J. MORAIS, R. M. PORTER, (2018), *Contragenic functions on spheroidal domains,* Mathematical Methods in the Applied Sciences, Volume 41, Issue 7, 2575-2589.

[3] J. MORAIS, (2013), *A complete orthogonal system of Spheroidal Monogenics,* Journal of Numerical Analysis, Industrial and Applied Mathematics 6 (3-4), 105-119.

Analytical and numerical analysis of waveguides and photonic crystals using the SPPS method

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Due to the positive impact that optical fibers have had, the improvement of their design has been studied for their use in telecommunication networks. The new designs have better optical properties, which allow lower losses on data transmission than conventional fibers. For this reason, during the decade of 1980, periodic structures, of the order of micrometers were used to experiment transmission of data, these structures are known as photonic crystals.

The form in which a crystal lattice is repeated, determines its optical properties and therefore the way electromagnetic waves propagate through it which allows us to control the flow of light. The light propagation is governed by Maxwell's equations. Employing vector identities as well as the constitutive relations of the medium, the electromagnetic field is splitted; this leads to second order differential equations, commonly known as Sturm-Liouville type equations.

One of the most common types of photonic waveguides are hollow core fibers, also known as Bragg fibers, these types of fibers have a hollow core and alternate layers that surround the core. These layers are optical materials with a high refractive index that guide the light into the hollow core. The importance of having a hollow core, filled of air, is to reduce the influence of undesired phenomena like dispersion. In this work, Bragg fibers are analyzed through the application of the Spectral Parameter Power Series (SPPS) method for solving the resulting Sturm-Liouville equations.

The analysis performed includes variations in the number and width of the layers in a Bragg fiber, the refractive indexes and their dimensions, and some characteristics are approached as the group velocity and the waveguide dispersion.

Neumann series of Bessel functions representations for solutions of linear higher order differential equations

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A new representation for solutions of the following Cauchy problem

$$y^{(n)} + \sum_{k=2}^{n} p_k y^{(n-k)} = \omega^n y, \tag{1}$$

$$y(\omega, 0) = 1,\ y'(\omega, 0) = \omega,\ \ldots,\ y^{(n-1)}(\omega, 0) = \omega^{n-1}, \tag{2}$$

is obtained in terms of Neumann series of Bessel functions. The coefficients p_k, $k = 2, \ldots, n$, are complex-valued continuous functions on $[0, b]$, $0 < b < \infty$, $n > 2$. The result is based on a Fourier-Legendre series representation for the Borel transform of the solution with respect to the spectral parameter ω. Estimates for the coefficients and for the convergence of the representation are derived.

References

[1] V. V. Kravchenko, S. M. Torba, and R. Castillo-Pérez, "A Neumann series of Bessel functions representation for solutions of perturbed Bessel equations," *Applicable Analysis*, vol. 97, no. 5, p. 677–704, 2018.

[2] A. Leontiev, *Obobshchennyye ryady eksponent*. [*Generalized exponential series*]. Moscow: Nauka (in Russian), 1981.

Soliton theory and Hankel operators

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Solition theory and the theory of Hankel (and Toeplitz) operators have stayed essentially hermetic to each other. This talk is concerned with linking together these two very active and extremely large theories. On the prototypical example of the Cauchy problem for the Korteweg-de Vries (KdV) equation we demonstrate the power of the language of Hankel operators in which symbols are conveniently represented in terms of the scattering data for the Schrodinger operator associated with the initial data for the KdV equation. This approach yields short-cuts to already known results as well as to a variety of new ones (e.g. wellposedness beyond standard assumptions on the initial data) which are achieved by employing some subtle results for Hankel operators.

A method for practical solution of forward and inverse spectral problems on finite and infinite intervals

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A new approach is presented for solving the classical forward and inverse Sturm-Liouville problems on finite and infinite intervals. It is based on the Gel'fand-Levitan-Marchenko integral equations and recent results on the functional series representations for the transmutation (transformation) operator kernels [1-5]. Representations for solutions of the Sturm-Liouville equation are obtained possessing attractive features for practical computation. In particular, a new representation for so-called Jost solutions allows one to reduce all calculations related to spectral and scattering data to finite intervals instead of the half line or the whole line. This is the case of the spectral density function as well as that of the reflection coefficient in the scattering problem. In a sense this reduction trivializes the classical spectral and scattering problems on infinite intervals previously considered as numerically challenging.

Numerical methods based on the proposed approach for solving forward problems allow one to compute large sets of eigendata with a nondeteriorating accuracy. Solution of the inverse problems reduces directly to a system of linear algebraic equations. In the talk some numerical illustrations will be presented.

1. V. V. Kravchenko, L. J. Navarro and S. M. Torba, Representation of solutions to the one-dimensional Schrödinger equation in terms of Neumann series of Bessel functions. Applied Mathematics and Computation, v. 314 (2017) 173–192.

2. V. V. Kravchenko, Construction of a transmutation for the one-dimensional Schrödinger operator and a representation for solutions. Applied Mathematics and Computation, v. 328 (2018) 75–81.

3. V. V. Kravchenko, On a method for solving the inverse scattering problem on the line. Mathematical Methods in the Applied Sciences 42 (2019) 1321–1327.

4. V. V. Kravchenko, On a method for solving the inverse Sturm-Liouville problem. Journal of Inverse and Ill-Posed Problems, 27 (2019), 401-407.

5. B. B. Delgado, K. V. Khmelnytskaya, V. V. Kravchenko, A representation

for Jost solutions and an efficient method for solving the spectral problem on the half line. Mathematical Methods in the Applied Sciences, to appear.

6. B. M. Levitan, Inverse Sturm-Liouville problems, VSP, Zeist, 1987.

7. V. A. Marchenko, Sturm-Liouville Operators and Applications. Birkhäuser, Basel, 1986.

8. V. A. Yurko, Introduction to the theory of inverse spectral problems. Moscow, Fizmatlit, 2007, 384pp. (Russian).

On the Green's function of a quantum waveguide with perturbations

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(Joint work with Víctor Barrera-Figueroa and Vladimir S. Rabinovich)

Let us consider the propagation of quantum waves in a layered waveguide consisting of a core between two layers that form the cladding (see, *e.g.*, [1]). The boundaries of the core are defined by two smooth functions $\phi_\pm(\varepsilon y)$ depending on a small parameter $\varepsilon > 0$, where y is the longitudinal (horizontal) coordinate of the waveguide [2]. The quantum waves generated by a point source are described by the boundary-value problem

$$\left(-\frac{\partial^2}{\partial y^2} + \mathcal{L}\left(y, z, \frac{\partial}{\partial z}\right) - E\right) G(y, z; y', z') = -\delta(y - y')\,\delta(z - z'),$$

$$[G(y, z; y', z')]_{z=\phi_\pm(\varepsilon y)} = 0, \qquad \left[\frac{\partial G}{\partial z}(y, z; y', z')\right]_{z=\phi_\pm(\varepsilon y)} = 0,$$

where $\mathcal{L}\left(y, z, \frac{\partial}{\partial z}\right) := -\frac{\partial^2}{\partial z^2} + V(y, z)$ is a vertical Schrödinger operator depending on the vertical coordinate z with y as a parameter, the potential V is defined by the piecewise continuous function

$$V(y, z) = \begin{cases} 0, & -\infty < z < \phi_-(\varepsilon y), \\ V_0(y, z), & \phi_-(\varepsilon y) < z < \phi_+(\varepsilon y), \\ 0, & \phi_+(\varepsilon y) < z < \infty, \end{cases}$$

being V_0 a real-valued function representing a quantum well, and E is the energy of the system. The operator $\mathcal{L}\left(y, z, \frac{\partial}{\partial z}\right)$ has a complete family of normalised eigenfunctions $\psi_j(z,y)$, and two families of normalised generalised eigenfunctions $\{\varphi_{01}(z,y;\alpha), \varphi_{02}(z,y;\alpha)\}_{\alpha \geq 0}$, where y acts as a parameter.

This talk is devoted to the construction of a modal representation of the Green's function G in terms of the eigenfuncitons ψ_j of the vertical Schrödinger operator $\mathcal{L}\left(y, z, \frac{\partial}{\partial z}\right)$ in the case when the boundaries $\phi_{\pm}(\varepsilon y)$ vary slowly in the longitudinal direction. In the limit as $\varepsilon \to 0$ the WKB (Wentzel-Kramers-Brillouin) [4] approach provides the sought modal expansion, and the SPPS (Spectral Parameter Power Series) method [3] provides exact solutions for the eigenfunctions ψ_j.

References

[1] Barrera-Figueroa V, Kravchenko V V, Rabinovich V S. Spectral parameter power series analysis of isotropic planarly layered waveguides. Appl. Anal. **93**: 729–755, 2014.

[2] Belov V V, Dobrokhotov S Y, Tudorovskiy T Y. Operator separation of variables for adiabatic problems in quantum and wave mechanics. J. Engineering Math. **55**: 183–237, 2006.

[3] Kravchenko V V, Porter R M. Spectral parameter power series for Sturm-Liouville problems. Math. Method App. Sci. **33**: 459-468, 2010.

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Approximation of Band-limited Functions by Quaternionic Ball Prolate Spheroidal Wave Signals

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In this talk, we discuss the energy concentration problem of band-limited quaternionic signals restricted in the spatial and frequency domains to the ball. Various structural results relating to the energy extremal properties between the spatial and frequency domains involving quaternionic signals are considered in detail. Key to the analysis are certain Quaternionic Ball Prolate Spheroidal Wave Functions, which possess a number of special properties that make them most useful for the study of band-limited functions.

Runge theorem for solutions of $(D + M^{\alpha})\varphi = 0$

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Let $\Omega \subseteq \mathbb{R}^3$ be a domain with connected complement. Regard $\mathbb{R}^3 \cong \{0\} \times \mathbb{R}^3 \subseteq \mathbb{H} = \{x_0 + x_1e_1 + x_2e_2 + x_3e_3\}$ with the gradient operator $D = \sum_1^3 e_i\,\partial/\partial x_i$. Let $\vec{\alpha} = (Df)/f \colon \Omega \to \mathbb{R}^3$ where $f \in C^1(\Omega, \mathbb{R})$ is a nonvanishing function. Suppose that f is separable in the sense that $f(x) = f_1(x_1)f_2(x_2)f_3(x_3)$.

We show that every quaternionic solution φ of $D\varphi + \varphi\vec{\alpha} = 0$ in Ω is a uniform limit (on compact subsets) of $\vec{\alpha}$-polynomials, that is, images of polynomial solutions of $D\varphi = 0$ under a suitable transmutation operator.

Spectral analysis of one-dimensional Schrödinger operators with two point interactions

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(Joint work with Víctor Barrera-Figueroa and Vladimir S. Rabinovich)

Let us consider the one-dimensional Schrödinger operator

$$S_q = -\frac{d^2}{dx^2} + q(x), \qquad q(x) := q_r(x) + q_s(x), \quad x \in \mathbb{R},$$

where $q_r \in \mathcal{L}^{\infty}(\mathbb{R})$ is a regular potential, and $q_s \in \mathcal{D}'(\mathbb{R})$ is a singular potential with support at $\{h_0, h_1\}$ defined by

$$q_s(x) = \alpha_0\delta(x - h_0) + \beta_0\delta'(x - h_0) + \alpha_1\delta(x - h_1) + \beta_1\delta'(x - h_1)$$

where $\alpha_i, \beta_i \in \mathbb{R}$, $i = 0, 1$.

As an unbounded operator in $\mathcal{L}^2(\mathbb{R})$ a domain of S_q must consist of functions $u \in \mathcal{L}^2(\mathbb{R})$ such that $S_q u \in \mathcal{L}^2(\mathbb{R})$. If $u \in \mathcal{C}_0^{\infty}(\mathbb{R} \setminus \{h_0, h_1\})$ this condition is satisfy but not for an arbitrary $u \in \mathcal{L}^2(\mathbb{R})$. However, it is possible to define an extension $\mathcal{H}$ of operator S_q that can act on discontinuous functions at $x = h_0$ and $x = h_1$. This extension is defined by the differential expression $S_{q_r} u := -u'' + q_r u$ in $\mathbb{R} \setminus \{h_0, h_1\}$ with domain

$$\text{Dom } (\mathcal{H}) = \left\{ u \in \widetilde{H}^2(\mathbb{R}) : \begin{pmatrix} u\left(h_i^+\right) \\ u'\left(h_i^+\right) \end{pmatrix} = \mathrm{A}_i \begin{pmatrix} u\left(h_i^-\right) \\ u'\left(h_i^-\right) \end{pmatrix}, \, i = 0, 1 \right\},$$

where

$$\mathrm{A}_i := \begin{pmatrix} \frac{4-\alpha_i\beta_i}{4+\alpha_i\beta_i} & -\frac{\beta_i}{4+\alpha_i\beta_i} \\ \frac{\alpha_i}{4+\alpha_i\beta_i} & \frac{4-\alpha_i\beta_i}{4+\alpha_i\beta_i} \end{pmatrix}, \, \alpha_i\beta_i \neq -4,$$

$\widetilde{H}^2(\mathbb{R}) = H^2(\mathbb{R} \setminus [h_0, h_1]) \oplus H^2(h_0, h_1)$, and $H^2(a, b)$ is Sobolev space in (a, b).

In previous expression we denoted by $u\left(h_i^{\pm}\right)$ and $u'\left(h_i^{\pm}\right)$ the one-sided limits of u and u' at h_i, $(i = 0, 1)$, respectively. Observe that matrices A_i thus defined satisfy $\det \mathrm{A}_i = 1$ $(i = 0, 1)$. If q_r is real-valued, then $\mathcal{H}$ is a self-adjoint operator in $\mathcal{L}^2(\mathbb{R})$.

In this talk we obtain the self-adjoint extension $\mathcal{H}$ of formal operator S_q and determine its discrete spectrum in $\mathcal{L}^2(\mathbb{R})$ for an (almost) arbitrary regular potential q_0. The discrete spectrum $\mathrm{sp}_{\mathrm{dis}}\mathcal{H}$ of operator $\mathcal{H}$ is obtained from the zeros of certain characteristic equation, which is explicitly constructed by means of the SPPS method [1].

References [1] Kravchenko VV, Porter RM. *Spectral parameter power series for Sturm-Liouville problems*. Math Methods Appl Sci. 2010; **33** (4): 459–468.

Perturbed Bessel equation with several spectral parameters

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A solution u_1 for the perturbed Bessel equation

$$-u''(x) + \big(\frac{l(l+1)}{x^2} + q(x)\big)u = \big(\sum \lambda_i r_i(x)\big)\, u(x),$$

$x \in [0, a)$, is described as a power series in the finitely many spectral parameters λ_i, with the asymptotics

$$u_1(x) \sim x^{l+1}, \; u_1'(x) \sim (l+1)x^l$$

at $x = 0$. The coefficient of $\lambda_1^n \cdots \lambda_d^n$ is obtained by summing appropriate combinations of iterated integrals using the data functions $q, r_1, \ldots, r_d$ and a nonvanishing solution u_0 of the equation for $\lambda_1 = \cdots = \lambda_d = 0$.

One-dimensional Dirac operators with delta-interactions

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We consider the Dirac operator on $\mathbb{R}$ of the form

$$\mathfrak{D}u(x) = \left(J\frac{d}{dx} + Q + Q_s\right)u(x),\ x \in \mathbb{R}$$

where $u = \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix}$ is a two-dimensional distribution on $\mathbb{R}$,

$$J = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$Q(x) = \begin{pmatrix} p(x) + r(x) & q(x) \\ q(x) & -p(x) + r(x) \end{pmatrix}, \quad p, q, r \in L^\infty(\mathbb{R}),$$

$$Q_s(x) = \sum_{y \in \mathbb{Y}} \Gamma(y)\, \delta(x - y),$$

$\Gamma(y) = (\gamma_{ij}(y))_{i,j=1,2}$ is a $2 \times 2-$matrix with elements $\gamma_{ij}(y) \in l^\infty(\mathbb{Y}), i, j = 1, 2$, $\mathbb{Y} \subset \mathbb{R}$ is a finite or infinite discrete set.

We associate with the formal Dirac operator $\mathfrak{D}$ the unbounded operator D in the Hilbert space $L^2(\mathbb{R}, \mathbb{C}^2)$ defined by the Dirac operator $\mathfrak{D}_Q = J\frac{d}{dx} + Q$ with regular potential Q and the point interaction conditions

$$A(y)u(y + 0) = B(y)u(y - 0),\ y \in \mathbb{Y}$$

defined by the singular potential Q_s, where $A(y) = \frac{1}{2}\Gamma(y) - J, B(y) = -(\frac{1}{2}\Gamma(y) + J)$.

We give conditions for the operator D to be self-adjoint in $L^2(\mathbb{R}, \mathbb{C}^2)$. Moreover we study the Fredholm properties and the essential spectrum of unbounded operator D. We consider the band-gap structure of spectra of Dirac operator D with periodic regular and singular potentials, and the influence of slowly oscillating perturbations of regular potentials Q on the essential spectrum of periodic Dirac operators.

Method of Riesz potentials applied to solution to nonhomogeneous singular wave equations

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Riesz potentials are convolution operators with fractional powers of some distance (Euclidean, Lorentz or other) to a point. From application point of view, such potentials are tools for solving differential equations of mathematical physics and inverse problems. For example, Marcel Riesz used these operators for writing the solution to the Cauchy problem for the wave equation and theory of the Radon transform is based on Riesz potentials. In this article, we use the Riesz potentials constructed with the help of generalized convolution for solution to the wave equations with Bessel operators. First, we describe general method of Riesz potentials, give basic definitions, introduce solvable equations and write suitable potentials (Riesz hyperbolic B-potentials). Then, we show that these potentials are absolutely convergent integrals for some functions and for some values of the parameter representing fractional powers of the Lorentz distance. Next we show the connection of the Riesz hyperbolic B-potentials with d'Alembert operators in which the Bessel operators are used in place of the second derivatives. Next we continue analytically considered potentials to the required parameter values that includes zero and show that when value of the parameter is zero these operators are identity operators. Finally, we solve singular initial value hyperbolic problems and give examples.

Sampling, oversampling, and subsampling in de Branges spaces: recent results

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An important mathematical technique in signal analysis and image processing is sampling theory. Kramer-type formulae for sampling functions in reproducing kernel Hilbert spaces are well known. In this context, the theory of de Branges spaces (dB spaces) is used so that the nodes of interpolation in the sampling formulae are in the spectrum of selfadjoint extensions of difference and differential operators. Recent developments show that, by recurring to the property that dB subspaces of dB spaces are totally ordered by inclusion, the concepts of oversampling and subsampling are generalized for a wide range of dB spaces.

This work was done in collaboration with J. H. Toloza and A. Uribe.

Historical survey of transmutation theory: persons and results

S.M. SITNIK

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The lecture is focused on the history of transmutation theory, its ideas and mathematicians who created this field.

Transmutation theory is deeply connected with many applications in different fields of mathematics. The transmutation operators are applied in inverse problems via the generalized Fourier transform, the spectral function and the famous Levitan equation; in scattering theory the Marchenko equation is formulated in terms of transmutations; in spectral theory transmutations help to prove trace formulas and asymptotics for spectral function; estimates for transmutational kernels control stability in inverse and scattering problems; for nonlinear equations via the Lax method, transmutations for Sturm-Lioville problems lead to proving existence and explicit formulas for soliton solutions. Special kinds of transmutations are the generalized analytic functions, generalized translations and convolutions, Darboux transformations. In the theory of partial differential equations, transmutations work for proving explicit correspondence formulas among solutions of perturbed and non-perturbed equations, for singular and degenerate equations, pseudodifferential operators, problems with essential singularities at inner or corner points, estimates of solution decay for elliptic and ultraelliptic equations. In function theory transmutations are applied to embedding theorems and generalizations of Hardy operators, Paley-Wiener theory, generalizations of harmonic analysis based on generalized translations. Methods of transmutations are used in many applied problems: investigation of Jost solutions in scattering theory, inverse problems, Dirac and other matrix systems of differential equations, integral equations with special function kernels, probability theory and random processes, stochastic random equations, linear stochastic estimation, inverse problems of geophysics and transsound gas dynamics. Also the number of applications of transmutations to nonlinear equations is permanently increasing.

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Integral Transform Composition Method (ITCM) in transmutaion theory: how it works

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In the lecture we study applications of integral transform composition method (ITCM) [1–4] for obtaining transmutations via integral transforms. It is possible to derive wide range of transmutation operators by this method. Classical integral transforms are involved in the ITCM as basic blocks, among them are Fourier, sine and cosine–Fourier, Hankel, Mellin, Laplace and some generalized transforms. The ITCM and transmutations obtaining by it are applied to deriving connection formulas for solutions of singular differential equations and more simple non–singular ones. We consider well–known classes of singular differential equations with Bessel operators, such as classical and generalized Euler–Poisson–Darboux equation and the generalized radiation problem of A.Weinstein. Methods of this paper are applied to more general linear partial differential equations with Bessel operators, such as multivariate Bessel–type equations, GASPT (Generalized Axially Symmetric Potential Theory) equations of

A.Weinstein, Bessel–type generalized wave equations with variable coefficients,ultra B–hyperbolic equations and others. So with many results and examples the main conclusion of this paper is illustrated: ***the integral transform composition method (ITCM) of constructing transmutations is very important and effective tool also for obtaining connection formulas and explicit representations of solutions to a wide class of singular differential equations, including ones with Bessel operators.***

In transmutation theory explicit operators were derived based on different ideas and methods, often not connecting altogether. So there is an urgent need in transmutation theory to develop a general method for obtaining known and new classes of transmutations. The ITCM gives the algorithm not only for constructing new transmutation operators, but also for all now explicitly known classes of transmutations, including Poisson, Sonine, Vekua–Erdelyi–Lowndes, Buschman-Erdelyi, Sonin–Katrakhov and Poisson–Katrakhov ones, as well as the classes of elliptic, hyperbolic and parabolic transmutation operators introduced by R. Carroll, cf. [1–4].

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An improved NSBF representation for regular solution of perturbed Bessel equations

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In [1] a Neumann series of Bessel function (NSBF) representation was proposed for the regular solu tion of a perturbed Bessel equation

$$-y'' + \frac{\ell(\ell+1)}{x^2} y + q(x)y = \omega^2 y, \qquad x \in (0, b].$$

Namely, for the solution $y_\ell(\omega, x)$ normalized by the asymptotic condition $y_\ell(\omega, x) \sim x^{\ell+1}$, $x \to 0$, the following representation

$$y_\ell(\omega, x) = \frac{2^{\ell+1}\Gamma\left(\ell + \frac{3}{2}\right)}{\sqrt{\pi}\omega^\ell} x j_\ell(\omega x) + \sum_{n=0}^{\infty} (-1)^n \beta_n(x) j_{2n}(\omega x),$$

was obtained, jointly with convenient formulas for the coefficients β_n.

However, this representation possesses two disadvantages:

- Saturation for non-integer values of ℓ: the coefficients β_n decay at most as $n^{-2\ell-3}$ independently of the smoothness of the potential q.
- Partial sums of the series provide good approximation only for small values of ω. This is due to the fact that $y_\ell(\omega, x)$ decays as $\omega^{-\ell-1}$ as $\omega \to \infty$.

In this talk we construct a NSBF representation of the form

$$\tilde{y}_\ell(\omega, x) = \sum_{n=0}^{\infty} \tilde{\beta}_n(x) j_{\ell+2n}(\omega x),$$

for the regular solution $\tilde{y}_\ell(\omega, x) := \omega^\ell y_\ell(\omega, x)$ and show how the mentioned disadvantages are avoided for this new representation.

The talk is based on joint work with V. V. Kravchenko.

[1] R. Castillo-Pérez, V. V. Kravchenko and S. M. Torba, "A Neumann series of Bessel functions representation for solutions of perturbed Bessel equations", Appl. Anal. 97 (2018), 677–704.

Research of dispersion problem in quantum waveguides by means of spectral parameter power series

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Main results of the study of dispersion problem in one-dimensional and three-dimensional quantum waveguides will be presented: the transmission and reflection coefficients for the scattering of a particle on one-dimensional potential are calculated by means of Spectral Parameter Power Series(SPPS); numerical results will be compared with known results. After, will be treated the electron propagation in a three-dimensional quantum waveguide which is described by the Dirichlet problem for the Schrödinger operator. A numerical algorithm will be presented to the calculation of the left and right transition matrices based on the SPPS method.

Non-Linear Fourier transform using transmutation operators and SPPS representations

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In this talk (joint work with V. V. Kravchenko and S. M. Torba) we focus on the direct non-linear Fourier transform for the Non Linear Schrödinger Equation, which reduces to the study of the Zakharov-Shabat (Z-S) system [1,2,3] of the form

$$\begin{pmatrix} v_1'(x) \\ v_2'(x) \end{pmatrix} = \begin{pmatrix} -i\lambda & q(x) \\ -q^*(x) & i\lambda \end{pmatrix} \begin{pmatrix} v_1(x) \\ v_2(x) \end{pmatrix},$$

where $v_{1,2}$ are unknown complex functions, λ is the spectral parameter, the complex-valued function $q(x)$ is the potential, $*$ is the complex conjugation and i is the imaginary unit. Since the Z-S system reduces to Sturm-Liouville equations we show, under a few restrictions for the potential, the spectral parameter power series [4] and the analytic approximation of transmutations operators [5] representations for the solutions of the Z-S system and the corresponding nonlinear Fourier coefficients. Finally we show numerical experiments, properties and numerical advantages of each method.

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Transmutation operators and complete systems of solutions for the d-dimensional radial Schrödinger equation

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The aim of this talk is to present an application of transmutation operators to the study of the radial Schrödinger equation. We consider in a star shaped domain Ω of the Euclidean space $\mathbb{R}^d$ (with dimension $d \geq 2$) the equation of the form

$$Su(x) := (\Delta_d - q(\|\mathbf{x}\|))u(\mathbf{x}) = 0 \quad \text{for } \mathbf{x} \in \Omega, \tag{1}$$

where q is a C^1 function that only depends of the radial component $r = \|x\|$. The transformation operator

$$Tf(\mathbf{x}) = f(\mathbf{x}) + \int_0^1 \sigma^{d-1} G(r,t) f(\sigma^2 \mathbf{x})\, d\sigma \tag{2}$$

has the property of transforming harmonic functions in Ω into solutions of (1) [1]. We show that actually (2) is a **transmutation operator** for $\widehat{S} := r^2 S$ and $\widehat{L} := r^2 \Delta_d$ in the space space $C^2(\Omega)$. Moreover, T is bounded in the Bergman space $b_2(\Omega)$ of square-integrable harmonic functions, and is a bijection between $b_2(\Omega)$ and the Bergman space $\mathfrak{S}_2(\Omega)$ of square-integrable solutions of (1).

Employing the properties of T, we obtain an orthogonal complete system of solutions $\mathcal{S} \subset \mathfrak{S}_2(\Omega)$ for (1), that generalizes the concept of harmonic homogeneous polynomial. The completeness of the system $\mathcal{S}$ in the sense of the uniform convergence in compact subsets of Ω, and in the L_2-norm, will be shown, with an explicit construction of the functions of $\mathcal{S}$.

Finally, we study an application to the solution of the Dirichlet problem in the unit ball $\mathbb{B}^d$.

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