
Center for Research and Advanced Studies of the IPN

Department of Mathematics

Admission Exam

April 17, 2026

(3 hours)

Instructions:

- Correctly solve at least 3 complete problems from each topic 1 .
- Justify all your answers 1 .

I. Linear Algebra

1. Let V be a finite-dimensional real vector space and let $A : V \rightarrow V$ be a linear operator such that $A^2 = I$ 1 .
 - (a) Prove that the only possible eigenvalues of A are 1 and -1 1 .
 - (b) Prove that $V = \ker(A - I) \oplus \ker(A + I)$ 1 .
 - (c) Conclude that A is diagonalizable over $\mathbb{R}$ 1 .
2. Prove that if A is upper triangular, then $\det(A) = \prod_{i=1}^n A_{i,i}$ 2 .
3. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation given by:
 $T(x, y, z) = (x + y + z, x + 2y + 3z, x + ay + 5z)$, where $a \in \mathbb{R}$ 2 .
 - (a) Determine for which values of a the transformation T is invertible 2 .
 - (b) For the values of a where T is not invertible, find a basis for $\ker T$ and a basis for $\text{Im} T$ 2 .
4. Let A be a square matrix over a field and A^t its transpose. Is it true (prove or provide a counterexample) that A and A^t are similar (i.e., there exists an invertible matrix C such that $A^t = CAC^{-1}$)? 2
5. Let $W \subset \mathbb{R}^4$ be the subspace spanned by the vectors:
 $v_1 = (1, 2, 3, 4), v_2 = (1, 1, 1, 1), v_3 = (-1, 2, -1, -2)$ 3 .
Find the orthonormal set that spans W 3 .

II. Calculus

1. Among all the points on the ellipse $\frac{x^2}{4} + y^2 = 1$, determine which points are the closest and furthest from the point $(1, 0)$ 3 .
2. Find the following limit (justify your answer) 3 :
$$\lim_{n \rightarrow \infty} \sum_{j=1}^n \frac{1}{(2j-1)^2}$$
3. Consider the sequence of functions $f_n(x) = nx^n(1-x)$, $x \in$ 4 .
 - (a) Study the pointwise convergence of (f_n) 3 .
 - (b) Determine if the convergence is uniform on 4 .
 - (c) Calculate $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx$ 5 .
4. Determine the values of the real parameter α for which the following improper integral converges:
$$\int_0^\infty x^\alpha (\sqrt{x^3+1} - x^{3/2}) \ln(1+x^2) dx$$
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5. If $f'(x) > 0$ for all x in an interval, prove that f is increasing on that interval 5 .