Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

February 11, 2002

Linear Algebra

1.1 What values for a in the following system:

a) have no solutions
b) have accurately a solution,
c) have countless solutions

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

1.2 Let $M_n(\mathbb{R})$ is the vector space of all real matrices n x n and let V be the subspace $M_n(\mathbb{R})$ that is made of all matrices of outline zero.

a) Calculate the dimension of V
b) Find the basis for V.

1.3 Let $v_1 = (1, -4, 7)$, $v_2 = (2, 5, -8)$ and $v_3 = (3, 6, 9)$ be 3 vectors in $\mathbb{R}^3$. Use the Gram-Schimidt process to find an orthonormal basis of $\mathbb{R}^3$ from v1, v2 and v3.

2. Calculus

2.1 Consider the function $F : [0, +\infty) \longrightarrow \mathbb{R}$ given by

$$F(x) = \int_0^x t^2 e^{t^2} dt$$

a) Find the continuity points of F.
b) Which of those points is F differentiable?
c) Calculate F'(2002)

2.2 Let x be a different positive real number of 1 and P is a prime number. Mention the cases where the following series are convergent:

$$\sum_{n=0}^{\infty} \frac{1}{x^{np}}.$$

2.3 Let $K \in \mathbb{R}^3$ the ellipsoid given by the equation $\frac{x2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ when a, b, c > 0. Given an arbitrary point of $(x, y, z) \in K$ in the first octant, consider the parallelepiped of vertices $(\pm x, \pm y, \pm z)$ inscribed in K, with volume V = 8xyz. Find the maximum value of V. Suggestion: V is the maximum value if and only if V^2 is the maxima value.

3. Optional problems

3.1 For $n \geq 1$, let $D^n = \{x \in \mathbb{R}^n | \|x\| \leq 1\}$ be the unitary disk in $\mathbb{R}^n$ and denoted by S^{n-1} to its borderline ∂D^n. Prove that $D^n/\partial D^n$ is homomorphical to S^n.

3.2 Give an example of an infinite group G but such that all its torsion elements.Find the image of the real straight line under the transformation $f : \mathbb{C} \longrightarrow \mathbb{C}$ given by $f(z) = \frac{z-i}{z=i}$.