Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

February 26, 2001

1. Linear Algebra

1.1 Determine that the real values a, b the following matrix is diagonizable on the real numbers:

$$A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

1.2 Let V be a subspace of $\mathbb{R}^4$ that is made of all the solutions to the following system of homogenous linear equations:

$$x + 3z + 2w = 0$$

$$x + y + w = 0$$

$$x + z = 0$$

Determine dimension V.

1.3 Find the orthonormal basis for the subspace of $\mathbb{R}^4$ generated by the vectors (1, 0, -1, 0), (1, 0, 1, 1) and (0, 0, 1, 1)

2. Calculus

2.1 Determine the values for the integer n, the following integral is finite:

$$\int_1^\infty x^n dx$$

2.2 Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function that possess first and second continuous derivatives. Prove that if $f''(x) > 0$ for each $x \in \mathbb{R}$ then f satisfies accurately one the following conditions:

1. f is strictly increasing
2. f is strictly decreasing
3. f posses a unique global minimum

2.3 Prove that among all the rectangles of fixed perimeter P the square of side P/4 possess the maximum area.

3. Optional Problems

3.1 Let K1 and K2 be two disjoint compact subsets of $\mathbb{R}^n$ prove that there are open disjoints U1 and U2 such that $k_1 \subset U_1$ and $K_2 \subset U_2$.

3.2 Let G be a finite group. Prove that if G/C(G) is cyclical (C(G)) denotes the center of G), then G is abelian.

3.3 Provide an example of a succession $(f_n)_n \subset L^1(\mathbb{R})$ such that $\lim_{n\longrightarrow\infty} f_n(x) = 0$ for each $x \in \mathbb{R}$ but that it meets $\int f_n = 1$ for each n.

3.4 Prove that there is a holomorphic mapping with holomorphic inverse

$f : \triangle \longrightarrow H$ where $\triangle = \{z \in \mathbb{C} | |z| = 1\}$ and $H = \{z \in \mathbb{C} | Im(z) > 0\}$.

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