Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

December 13, 2007

1. Linear Algebra

1.1 Let p, q, r and s be polynomials of grade greater than 2. Which of the following conditions, if there is one, is enough to conclude that polynomials are linearly dependant?

a) The value of polynomials evaluated is zero
b) The value of the polynomials evaluated in 0 is one.

1.2 Do real numbers r_1, r_2, r_3 and r_4 such as polynomials:

$$P_1(x) = (x - r_1)(x - r_2)$$
$$P_2(x) = (x - r_2)(x - r_3)$$
$$P_3(x) = (x - r_3)(x - r_4)$$
$$P_4(x) = (x - r_4)(x - r_1)$$

are linearly independent?

1.3 Suppose that A and B are endophormisms of a vector space V of a finite dimensional on a field F. Prove or provide an opposite example of the following statements:

a) Every eigen vector of AB is an eigenvector of BA
b) Every eigen value of AB is a eigen value of BA

2. Calculate:

2.1 Let $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, 0) = 0$ and

$$f(x,y) = (1 - \cos\frac{x^2}{y})\sqrt{x^2 + y^2} \quad \text{for} \quad y \neq 0.$$

a) Prove that f is continuous in (0, 0)
b) Calculate that all directional derivatives of f in (0, 0)
c) Prove that f is not differentiable in (0, 0)

2.2 Determine if the following series:

$$\sum_{n=1}^{\infty} \frac{3}{n^2} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{2^n}{n!}$$

converge and that explain why.

2.3 Calculate the following limit:

$$\lim_{x \to 1} \frac{x^3 - 2x + 1}{2x^3 - 3x^2 + 5x - 4}$$

3. Optional problems

3.1 Find all groups with eight elements.

3.2 Let X be a connected topological space and locally connected-arch. Prove that X is a connected arch.

3.3 Prove that for every x you have:

$$2^x + 3^x - 4^x + 6^x - 9^x \leq 1$$

3.4 Prove that if G is a group in which every element is it own reversal, then G is abelian.