Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

December 16, 2008

1. Linear Algebra

1.1 Let *A* be an squared matrix of order n with real entries and be *I* the identity matrix of order n. Prove that if $A^2 = 2I$ then A is an invertible matrix. Find the inverse for A in terms of *I* and *A*

1.2 Determine a matrix in respects of the canonical basis of a linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ that satisfies $T^2 = I$ y $T((1,1)) = (1,0)$.

1.3 Let A be a square matrix of order n with invertible real entries. Prove that there are real matrices P and Q such that P is positive defined symmetric, Q is orthogonal (that is, $QQ^t = I$) and A= PQ.

Suggestion: use the properties in AA^t

2. Calculus

2.1 Let *k* be a fixed positive integer and 0 < a < 1 a real number. Prove that the limit

$$\lim_{n\to\infty} \binom{n}{k} a^n = 0$$

Remember that $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.

2.2 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined for $f(x,0) = 0$ and

$f(x,y) = (1 - \cos\frac{x^2}{y})\sqrt{x^2+y^2}$ for $y \neq 0.$

a) Prove that ***f*** is continuous in (0,0)
b) Calculate all directional derivatives of ***f*** in (0,0)
c) Prove that ***f*** is not differentiable in (0, 0)

2.3 Prove that the equation $ae^x = 1+x+\frac{x^2}{2}$ where ***a*** is a positive constant, has exactly a real root.

3. Optional Problems

3.1 Provide an example or prove that there are no examples for each of the following groups:

a) One non-abelian group
b) One finite non-cyclic abelian group
c) One finite group with subgroups of index five
d) Two finite non-isomorphic groups of the same order
e) One group of G with a non-normal subgroup H
f) One group G with a non-normal subgroup H of index two

3.2 Let R be the set of real numbers with usual topology. Which of the following statements are true?

a) The union of the whole family of open sets is an open set
b) The union of the whole family of closed sets is a closed sets
c) The whole infinite and bounded set has a succession of unique points that converge in R.

3.3 Let f be a continuous function on [0, 1]. Calculate the following limit:

$$\lim_{n\to\infty} \int_0^1 x^n f(x) dx.$$

3.4 Prove that for each x you have:

$$2^x + 3^x - 4^x + 6^x - 9^x \leq 1$$