Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

August 17, 1998

1. Linear Algebra

1.1 Consider the matrix:

$$A = \begin{pmatrix} 2 & -1 & 7 & 1 \\ 1 & 2 & -4 & 1 \\ 1 & 0 & 2 & 1 \end{pmatrix}$$

Find the basis for the image of the linear transformation $T : \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ defined by A.

1.2 Consider the matrix:

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

Determine appropriate values for A and a basis for subspaces of corresponding appropriate values.

1.3 Use elementary operations to determine the matrix inverse.

$$A = \begin{pmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

2. Calculus

2.1 Say if the following series converge or not and why.

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \quad y \qquad \sum_{n=0}^{\infty} \frac{2^n}{n!}$$

2.2 Find the derivative of the function *F* on [0, 1] like:

$$(a)\ F(x) = \int_0^x (\sin t^2\ )dt,$$

$$(b)\quad F(x) = \int_0^{x^2} (1+t^3)^{-1} dt.$$

2.3 Graph the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by $f(x) = x^3 - 3x$ noting local extremes, inflexion point and intervals in concavity and convexity

3. Optional problems

3.1 Provide a non-differential function $F : \mathbb{R}^2 \longrightarrow \mathbb{R}$ in (0, 0) which partial derived exist in (0, 0)

3.2 Let $\{Fi\}_{i=1}^{\infty}$ be a succession of closed sets in $\mathbb{R}^n$. is $\bigcup_{i=1}^{\infty} F_i$ a closed set in $\mathbb{R}^n$.

3.3 Let A be a matrix of order n. If $A^t = -A$ and n is even, prove that A = 0. Remember that At denotes transpose of A.

3.4 Let (X_1, d_1) and (X_2, d_2) be metric spaces and be $X = X_1 \times X_2$ (the Cartesian product). Prove that the function $d : X \longrightarrow \mathbb{R}$ defined by:

$$d((x_1, x_2),(x_1', x_2'))= d_1(x_1, x_1')+d_2(x_2, x_2')$$

is metric in X.

3.5 Let $(\mathbb{Z}_n, +)$ be the additive group of the integers in module n. Is the Cartesian product $\mathbb{Z}_2 \times \mathbb{Z}_4$ isomorphic to $\mathbb{Z}_8$?