ON ANALYTIC TYPE FUNCTION SPACES AND DIRECT SUM DECOMPOSITION OF $L_2(D, d\nu)$

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Let D be either the unit disk $\mathbb{D}$ or $\mathbb{C}$, and let J be either $[0,1)$ or $\mathbb{R}_+$, so that $D = J \times \mathbb{T}$, where $\mathbb{T}$ is the unit circle in $\mathbb{C}$. We set $\mathcal{H}$ for any weighted Hilbert space $L_2(D, d\nu)$, with the probability measure $d\nu(z) = \omega(|z|)dA(z)$, where $dA(z) = \frac{1}{\pi}dxdy$, $z = x + iy$, and whose radial weight function $\omega : D \to \mathbb{R}_+$ is such that the linear span of the monomials $z^p\overline{z}^q$, for all p, $q \in \mathbb{Z}_+$, is dense in $\mathcal{H}$.

Given any pair $(m,n) \in \mathbb{Z}_+ \setminus \{(0,0)\}$, we denote by $\mathcal{A}^{(m,n)}$ the subspace of $\mathcal{H}$, which consists of all smooth functions f satisfying the equation $\frac{\partial^m}{\partial z^m}\frac{\partial^n}{\partial \overline{z}^n} f = 0$, and by $\mathcal{A}_k^{(m,n)}$ the subspace of $\mathcal{H}$, which consists of all smooth functions satisfying the equation $\left(\frac{\partial^m}{\partial z^m}\frac{\partial^n}{\partial \overline{z}^n}\right)^k f = 0$.
We call such functions (m,n)-*analytic*, and k-(m,n)-*polyanalytic*, respectively.

For particular values of (m,n), we have already known spaces of
- analytic functions $\mathcal{A} = \mathcal{A}^{(0,1)}$,
- k-polyanalytic functions $\mathcal{A}_k = \mathcal{A}^{(0,k)}$,
- anti-polyanalytic functions $\widetilde{\mathcal{A}} = \mathcal{A}^{(1,0)}$,
- k-anti-polyanalytic functions $\widetilde{\mathcal{A}}_k = \mathcal{A}^{(k,0)}$,
- harmonic functions $\mathsf{H} = \mathcal{A}^{(1,1)}$,
- k-polyharmonic functions $\mathsf{H}_k = \mathcal{A}^{(k,k)}$.

We develop a unified approach to the characterization of all these analytic type function spaces and prove, in particular, the following result.

Given any predefined "analytic quality of functions", $(m,n) \in \mathbb{Z}_+ \setminus \{(0,0)\}$, the Hilbert space $L_2(D, d\nu)$ admits the following direct sum decomposition

$$L_2(D, d\nu) = \bigoplus_{k\in\mathbb{N}} \mathcal{A}_{(k)}^{(m,n)},$$

where $\mathcal{A}_{(k)}^{(m,n)} = \mathcal{A}_k^{(m,n)} \ominus \mathcal{A}_{k-1}^{(m,n)}$ are the spaces of the so-called true-k-(m,n)-polyanalytic functions.

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