Representaciones inducidas

lunes, 29 de noviembre de 2021
11:01 a. m.

Si G es un grupo finito y $H \leq G$

$\rho : H \to Gl(V)$

$\mathrm{Ind}_H^G V = \mathbb{K}[G] \otimes_{\mathbb{K}[H]} V \curvearrowleft G$

$\mathbb{K}(G)$ es el $\mathbb{K}$-esp vet con base G

Si $\rho: G \to Gl(V)$ $\quad H \overset{i}{\hookrightarrow} G$

$\mathrm{Res}_H^G V = V \qquad \rho \circ i : H \to Gl(V)$

$\mathrm{Hom}_G(W, \mathrm{Ind}\, V) = \mathrm{Hom}_H(\mathrm{Res}\, W, V)$

Sea G un grupo de Lie y $H \leq G$ cerrado

$\rho : H \longrightarrow GL(V)$

$\underline{\mathrm{Ind}_H^G V} = \mathrm{Map}_H(G, V) = \{ f : G \longrightarrow V \mid f(gh^{-1}) = hf(g) \ \forall h \in H \text{ y } g \in G$

$G \curvearrowright \mathrm{Ind}_H^G V \qquad (g \cdot f)(v) = f(\rho(g^{-1})v)$

$\varphi : G \longrightarrow GL(V) \Rightarrow \mathrm{Res}_H^G \rho : H \longrightarrow GL(V)$

Otra construcción de $\mathrm{Ind}_H^G V$

$$
\begin{tikzcd}
V \arrow[r, leftrightarrow] & G\times V/\sim \arrow[d, "p = pr_1"] \\
 & G/H
\end{tikzcd}
$$

$(g,v) \sim (gh^{-1}, h\cdot v) \quad h \in H$

id

G/H ← variedad suave

$$\Gamma(G\times V/\sim) = \{ s : G/H \to G\times V/\sim \mid p\circ s = \mathrm{id} \}$$

secciones de $G\times H/\sim$

$$\cong$$

$$\mathrm{Ind}_H^G V$$

Def. Un haz vectorial es una función continua tal que satisface cierta trivialidad local y cada fibra tiene la estructura de un $\mathbb{K}$ esp. ve

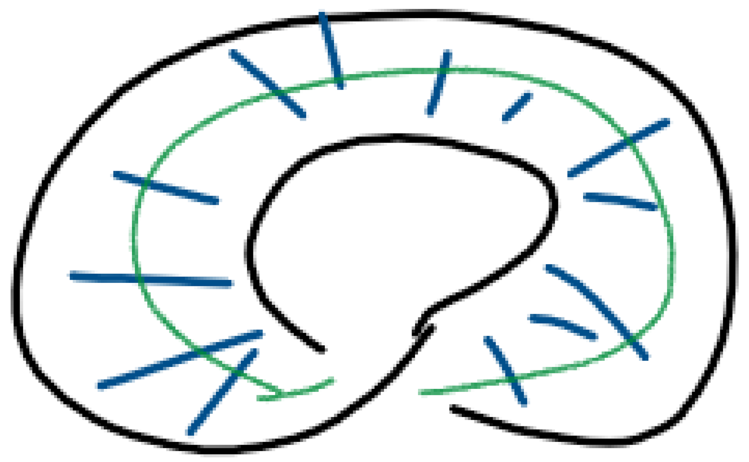
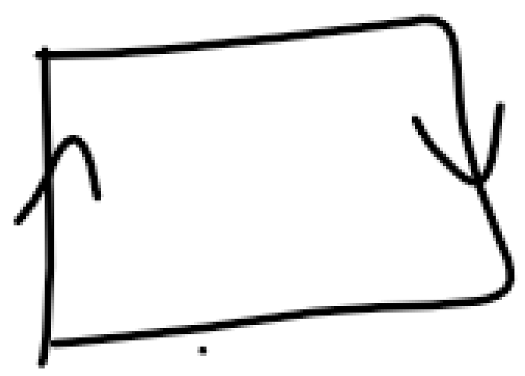
$$
\begin{tikzcd}
\text{fibra} \quad F \arrow[r, hook] & E \quad \leftarrow \text{total} \\
& x \in B \arrow[from=u, "p"] \quad \leftarrow \text{base}
\end{tikzcd}
$$

$F = p^{-1}(x)$

$\exists\, U \subset B \quad x \in U$ y tal que

$$
\begin{tikzcd}
p^{-1}(U) \arrow[rr, "\cong"] \arrow[dr, "p"'] & \circlearrowleft & U \times F \arrow[dl, "pr_1"] \\
& U &
\end{tikzcd}
$$

Ejemplos:

•) $\mathbb{R} \longrightarrow$  $\cong$ 

$\downarrow$

S^1

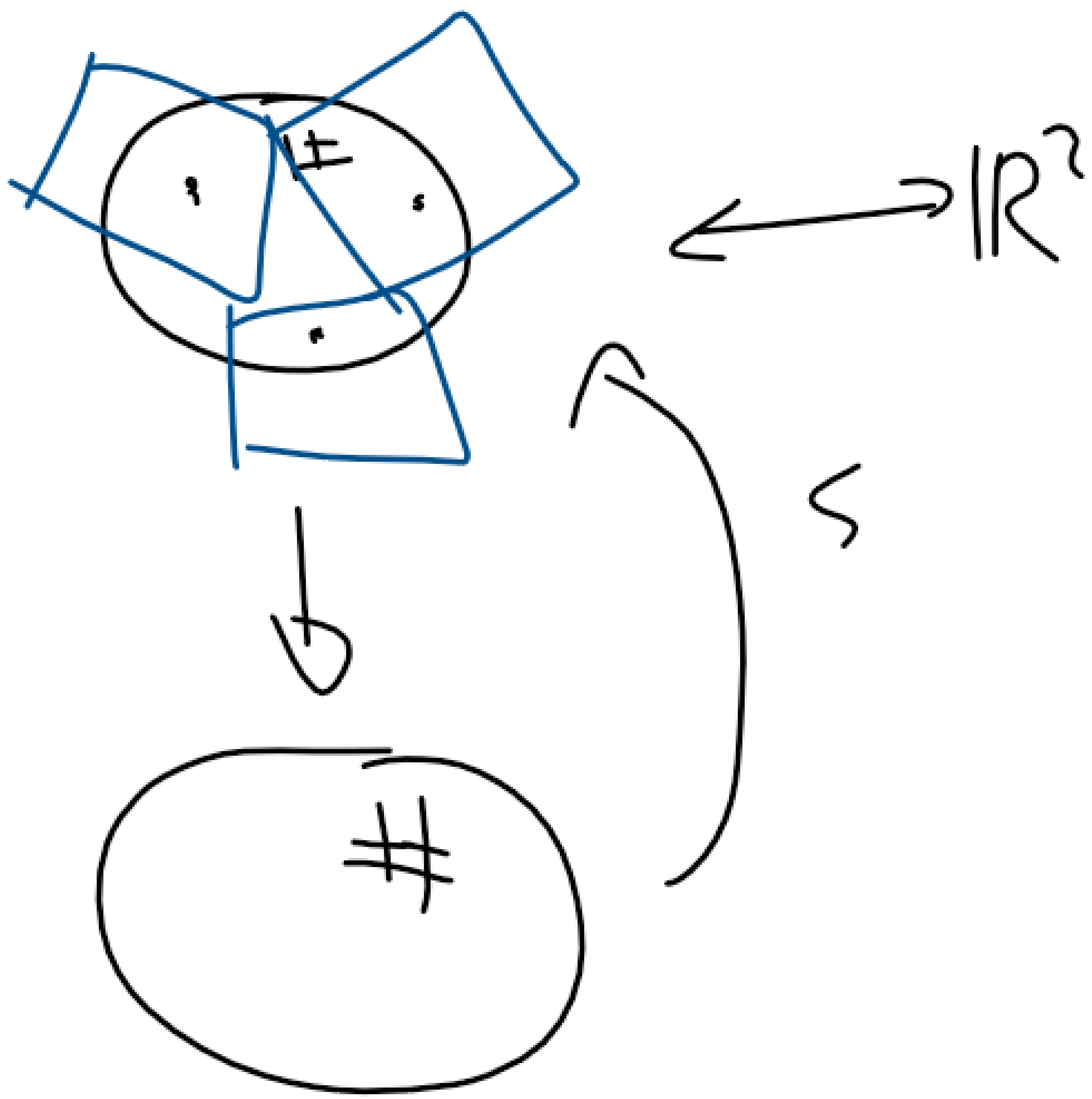
*)

M^n una variedad $\quad \mathbb{R}^n \longrightarrow TM := \{(x,v) \in M, \mathbb{R}^n \mid v \in T_x M$

$$\begin{array}{c} TM \\ \downarrow \\ M \end{array}$$

Haz tangente

$M = S^2$



Vect(M)

Teorema. Existe un isomorfismo de G rep.

$$\Gamma(G \times_H V) \xrightarrow{\varphi} \operatorname{Ind}_H^G V$$

Dem

$$g \cdot s(v) = s(g^{-1}v)$$

sea $f \in \operatorname{Ind}_H^G V = \{ f : G \to V \mid f(gh^{-1}) = hf(g) \; h \in H \; g \in$

$$\Gamma \ni s_f : G/H \longrightarrow G \times_H V$$

$$gH \longmapsto (g, f(g))$$

$$(gh, f(gh)) = (gh, h^{-1} f(g)) \sim (g, f(g))$$