Roots for n-valued maps.

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Given an integer n, an $n-$valued map $\varphi : X \multimap Y$ and a base point $a \in Y$ we define by $root(\varphi) = \{x \in X | a \in \varphi(x)\}$, which generalize the usual concept of root of single valued maps. The root of single valued maps has been extensively studied and there is a substantial number of results. Such subject is usually referred as Nielsen root theory. In this talk we will present an approach to study properties known for single valued in the case of n-valued maps. Notable we will focus in a result by Brooks for single valued maps which is as follow:
Let $f : X \to M$ be a continuous map into a closed manifold then all root classes are all essentials or are all inessential. We will see that a claim similar to the one above is not true for n-valued maps, but we will show that under certain hypotheses it holds for n-valued maps. To obtain the results we study the n-valued maps by divide them into two families, namely one which consist of the maps so called split and the ones which are not split. The hypotheses for which the result holds are given in terms of the fundamental group of $\pi_1(X)$, $\pi_1(D_n(Y))$ (where $D_n(Y)$ denote the n-th unordered configuration space of Y) and a homomorphism between these groups. Follows some references relevant to the development of this work.

References

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(*) Due to Bob Brown's recent passing, I am fully responsible for any errors or misinterpretation that may occur during this talk.