

An Overview of p-adic Algorithmic Fewnomial Theory

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ABSTRACT

Fewnomial Theory began with Rene Descartes' observation, around 1637, that real univariate polynomials with t monomial terms have no more than $t-1$ positive roots. It would not be until about 350 years later that Askold Khovanski would extend this bound to systems of real multivariate polynomial equations, and another 10 years before Hendrik Lenstra found a proper p-adic analogue of Descartes' bound (in one variable). However, a missing fact is precise bounds on the complexity of solving sparse polynomial equations. Indeed, while Paul Cohen found the first tractability results for deciding arbitrary sentences (in first order logic) over the p-adic rationals around 1969, the fine-grained complexity of solving equations remains quite open. We survey the latter, focusing on some definitive speed-ups in one variable.