p-Adic numbers, dynamical systems, and morphogenesis

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ABSTRACT

The matter is organized in various forms of complexity. The living organisms present it at the highest level, conjecturally due to an evolutionary process of more than 35,000 years. Patterns of complexity are observed at different levels of self-organization: macromolecules, cells, organisms, populations, ecosystems, etc. It has been studied from different disciplines and points of view, complementing the biological understanding: the physical, the chemical, the informatics, and the mathematical. Traditionally they have been studied with real analysis tools, differential equations, and classical dynamical systems. However, it is also possible to have an approach through p-adic analysis and non-Archimedean analysis in general, thus including number theory in biomathematics.

Morphogenesis is the biological process that leads an organism to develop its shape. In this talk, I will present the mathematical bases of p-adic dynamical systems and finite fields that have allowed us to study morphogenesis in flowers. First, I will define Epigenetic Forests as the transition graph of a dynamic system over a finite field associated with the Genetic Regulatory Network (GRR) of the plant Arabidopsis thaliana during cell fate determination. We also define a measure of the energy of specialization of undifferentiated cells and pose an optimization problem whose optimal solution manages to recover the architecture of the flower correctly. Later, we present how this model can be extended to a dynamical system over p-adic numbers and Berkovich spaces.