

# Radial symbols and Toeplitz operators on Cartan Domains of type I

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## Resumen

We denote by  $D_{n \times n}^I$  the Cartan domain of type I given by the matrices  $Z \in M_{n \times n}(\mathbb{C})$  that satisfy  $Z^*Z < I_n$ . For  $n = 1$  this trivially reduces to the unit disk  $\mathbb{D}$ . On the latter we have the so-called radial symbols given by the condition  $a(z) = a(|z|) = a(\sqrt{\bar{z}z})$ , for every  $z \in \mathbb{D}$ . On the domain  $D_{n \times n}^I$  we will consider symbols satisfying either of the conditions

$$a(Z) = a((Z^*Z)^{\frac{1}{2}}) \text{ or } b(Z) = b((ZZ^*)^{\frac{1}{2}})$$

for  $Z \in D_{n \times n}^I$ . We will show that, for  $n \geq 2$ , these two conditions are not equivalent. Furthermore, they define two families of symbols whose Toeplitz operators generate non-commutative  $C^*$ -algebras which centralize each other. A third condition is considered, which is

$$a(UZV) = a(Z),$$

for every  $U, V \in U(n)$  and  $Z \in D_{n \times n}^I$ . For any  $n$ , this condition does yield symbols whose Toeplitz operators generate commutative  $C^*$ -algebras. Note that for  $n = 1$ , all three conditions are equivalent and correspond to the definition of radial symbols. For  $n \geq 2$ , these conditions are all different and will be used to construct Banach non- $C^*$  algebras generated by Toeplitz operators.